Time Truncated Chain Sampling Plans for Generalized Exponential Distribution

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Abstract:
This paper introduces Chain sampling plan for generalized exponential distribution when the life-test is truncated at a pre-specified time are provided in this manuscript. It is assumed that the shape parameter of the generalized exponential distribution is known. The design parameters such as the minimum sample size and the acceptance number are obtained by satisfying the producer's and consumer's risks at the specified quality levels in terms of medians, under the assumption that the termination time and the number of items are pre-fixed.

Keywords: Truncated life test, Generalised exponential distribution, consumer's risk, Operating characteristics, Producer's risk, truncated life test.

1. Introduction

Quality control has become one of the most important tools to differentiate between the competitive enterprises in a global business market. Two important tools for ensuring quality are the statistical quality control and acceptance sampling. The acceptance sampling plans are concerned with accepting or rejecting a submitted lots of a size of products on the basis of quality of the products inspected in a sample taken from the lot. An acceptance sampling plan is a specified plan that establishes the minimum sample size to be used for testing.

In most acceptance sampling plans for a truncated life test, major issue is to determine the sample size from the lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in tester. Sampling inspection in which the criteria for acceptance and non acceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in Chain Sampling Plan.

The purpose of this study is to find the probability of Acceptance for chain sampling plan assuming the experiment is truncated at preassigned time and lifetime follows a generalized exponential distribution.

2. Generalised exponential distribution:

The two parameter generalized exponential distribution has the following probability density function (PDF);

$$f(x;\alpha,\lambda) = \frac{\alpha}{\lambda} e^{-\frac{x}{\lambda}} (1-e^{-\frac{x}{\lambda}})^{\alpha-1}; \quad x > 0 \text{ -------(1)}$$

Here $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters respectively. We use GE(α, λ) to denote generalized exponential random variable with the PDF (1)

The two-parameter generalized exponential distribution was originally introduced by Gupta and Kundu [7] can be used effectively in many circumstances which might fit better than Weibull or gamma distribution in some cases. It is observed that the shape of the PDF and (Hazard Function) of the generalized exponential distribution depend on the shape parameter α. The PDF is a decreasing function or an unimodal function if $0 < \alpha \leq 1$ or $\alpha > 1$ respectively. The Hazard Funtion of the generalized exponential distribution is a decreasing function if $\alpha < 1$ and for $\alpha > 1$ it is an increasing function. The PDF's and HF's of the generalized exponential distribution are very similar to those of Weibull and gamma distributions. In different studies it has been shown that for certain ranges of the parameter values, it is extremely difficult to distinguish between Generalised exponential and Weibull, gamma, log-normal, generalized Rayleigh distributions.

The cumulative distribution function (CDF) of GE(α, λ) is given by

$$F_{GE}(x;\alpha,\lambda) = (1- e^{\frac{x}{\lambda}})^{\alpha} .$$

If X ~ GE(α, λ) , then the mean and variance of X can be expressed as

E(X) = λ [$\psi(\alpha+1) - \psi(1)$] , V(X) = λ^2 [$\psi'(1) - \psi'(\alpha+1)$]

Here $\psi(.)$ and $\psi'(.)$ are digamma and polygamma functions respectively , i.e.

$$\psi(u) = \frac{d}{du}\Gamma(u),\ \psi'(u) = \frac{d}{du}\psi(u),\ \text{where } \Gamma(u) = \int_0^\infty x^{u-1}e^{-x}dx\,.$$

It is clear that both the mean and variance are increasing functions of λ. Therefore, the median of GE(α,λ) becomes;

$$\theta_m = -\lambda \,\text{In}\,(1- (\frac{1}{2})^{\frac{1}{\alpha}}).$$

From now on unless otherwise mentioned, we treat θ_m as the quality parameter. From the above equation it is clear that for fixed $\alpha = \alpha_0$, $\theta_m \geq \theta_m{}^0 \Leftrightarrow \lambda \geq \lambda_m{}^0$, where

$$\lambda_m{}^0 = \frac{\theta_m{}^0}{-In\left(1-\left(\frac{1}{2}\right)^{\frac{1}{\alpha_0}}\right)}.$$

One can see that $\lambda_m{}^0$ also depends on α_0. Now we develop the chain sampling plan for the generalized exponential distribution to ensure that the median lifetime of the items under study exceeds a predetermined quality provided by the consumer say θ_m ,equivalently λ exceeds $\lambda_m{}^0$, with a minimum probability P^*.

3. Design of the proposed sampling plan

Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) making use of cumulative results of several samples help to overcome the shortcomings of the Single Sampling Plan. It avoids rejection of a lot on the basis of a single nonconfirming unit and improves the poor discrimination between good and bad quality that occurs with the c = 0 plan.

The use of cumulative results of several samples is proposed for application to cases where there is repetitive production under the same conditions and where the lots or batches of products to be inspected are submitted for inspection in the order of production. Such situation may arise in receiving inspection of a continuing supply of purchased materials produced with in a manufacturing plan. Chain sampling is not suited to intermittent or job lot production , or to occasional purchases. An example situation is a continuing supply of processed material, such as a particular type of copper alloy rod .

When large samples are not practicable, and the use of c = 0 plan is warrented , for example, when an extremely high quality is essential the use of chain sampling plan is often recommended. The conditions for application and operating procedure of chsp-1 are as follows

3.1 Conditions for application of ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.

1) The product to be inspected comprises a series of successive lots produced by a continuing process.
2) Normally lots are expected to be of essentially the same quality.
3) The consumer has faith in the integrity of the producer.

3.2 Procedure:

Suppose n units are placed in a life test and the experiment is stopped at a predetermined time T. The number of failures till the time point T is observed, and suppose it is d. The decision to accept the lot takes place, if and only if the number of failures d at the end of the time point T does not exceed i =1 the acceptance number .

The plan is implemented in the following way:

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1.
3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceeding i samples of size n.

Dodge (1955) has given the operating characteristic function of ChSP-1 as $P_a(p) = P_0 + P_1 (P_0)^i$,
Where P_a = the probability of acceptance,

P_0 = probability of finding no defects in a sample of n units from product of quality p.

P_1 = probability of finding one defect in such a sample.

i = Number of preceding samples.

The Chain sampling Plan is characterized by the parameters n and i. When i= ∞,the OC function of a ChSP -1 plan reduces to the OC function of the Single Sampling Plan with acceptance number zero and when i = 0, the OC function of Chsp -1 plan reduces to the OC function of the Single Sampling Plan with acceptance number 1.

We are interested in determining the sample size required for in the case of generalized exponential distribution and various values of acceptance number i.

The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability(β) of accepting a bad lot is known as the consumer's risk. Often the consumer risk is expressed by the consumer's confidence level. If the confidence level is p* then the consumer's risk will be β = 1- p*.We will determine the sample size so that the consumer's risk does not exceed a given value β. The probability of acceptance in the case of chain sampling plan is given by

$$L(p) = (1-p)^n + np(1-p)^{n-1}(1-p)^{ni}$$

Where $p = F_{GE}(T;\alpha,\lambda) = (1-e^{-T/\lambda_m})^{\alpha}$

It is clear that p depends only on the ratio for a fixed $\alpha = \alpha_0$.

In Table 1 we present the minimum values of n $T/\lambda_m^{\ 0}$, satisfying equation

for p* = 0.75, 0.90, 0.95, 0.99 and for $T/\lambda_m^{\ 0}$ = 0.628,0.942,1.257,1.571,2.356,3.141,4.712, keeping α_0 fixed. These choices are consistent with Gupta and Groll (1961), Gupta (1962),

Kantam et al (2001),Baklizi and EI Masri (2004),Balakrishnan et Al (2007).

4. Operating Characteristic functions

The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function of the sampling plan. Once the minimum sample size is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. For a fixed T, For a fixed $\alpha = \alpha_0$ and i the operating chartacteristic function values as a function of $\lambda/\lambda_m^{\ 0}$ are presented in Table 2 for different values of p^*.

5. Notations

n	-	sample size
α, λ	-	shape parameters
i	-	Acceptance criteria
T	-	Termination time
θ_m	-	Quality parameter

d	-	Number of defectives
α	-	Producer's risk
β	-	Consumer's risk
p*	-	Minimum probability
L(p)	-	Probability of acceptance

6. Description of tables and examples

Assume that the life time distribution is an generalised exponential distribution with $\alpha_0 = 2$ and that the experimenter is interested in knowing that the true median life is atleast 1000 hours with confidence 0.99. It is assumed that the maximum affordable time is 767 hours .Since

$$\lambda_m^{\;0} = \frac{1000}{-In\left(1-\left(\frac{1}{2}\right)^{\frac{1}{2}}\right)} = 814.37$$

,

And $T/\lambda_m^{\;0} = 0.942$,

From the table 1, we obtain n = 10. Therefore, out of 10 items if not more than 1 item fail and if no defectives are found in the immediately preceeding i samples before T = 767 units of time,the lot can be accepted with the assurance that the true median life is atleast 1000 with probability 0.99.

For the sampling plan(n = 10, i= 2, $T/\lambda_m^{\;0} = 0.942$) and confidence level p* = 0.99 under generalised exponential distribution with $\alpha_0 = 2$ the values of the operating characteristic function from Table 2 as follows

λ/λ_0	2	4	6	8	10	12
L(p)	0.235645	0.756771	0.92156	0.96932	0.98593	0.99273

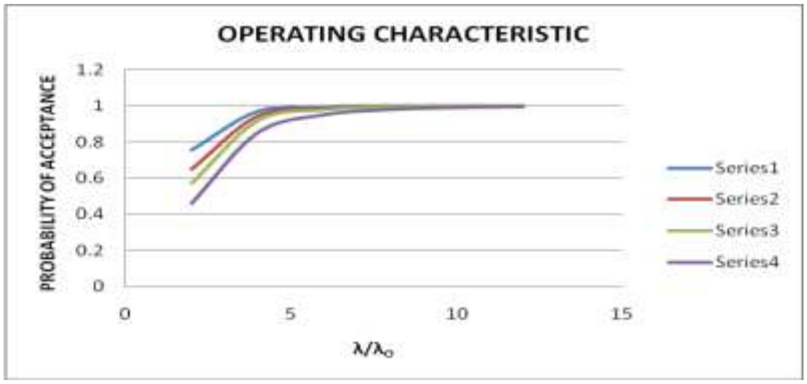

Figure 1. OC curve for Probability of acceptance against λ/λ_0

7. Conclusion

In this paper, chain sampling plan for the truncated life test was proposed in the case of generalized exponential distribution. It is assumed that the shape parameter is known , the minimum sample size required and the acceptance number were calculated. From the figure 1 we can see that the probability of acceptance increases when λ/λ_0 increases and it reaches the maximum value 1 when λ/λ_0 is greater than 5. This operating characteristic function values can be used for other distributions also which can be converted to generalized exponential distribution.

Table 1:

Minimum sample size for the proposed plan in case of exponential distribution with probability p* and the corresponding acceptance number i when the shape parameter $\alpha_0 = 2$.

p*	i	T/λ_m^0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	1	7	4	3	2	2	1	1	1
	2	6	4	3	2	1	1	1	1
	3	6	4	2	2	1	1	1	1
	4	6	3	2	2	1	1	1	1
	5	6	3	2	2	1	1	1	1
	6	6	3	2	2	1	1	1	1
0.90	1	11	6	4	3	2	2	1	1
	2	10	6	4	3	2	1	1	1
	3	10	5	4	3	2	1	1	1
	4	10	5	4	3	2	1	1	1
	5	10	5	4	3	2	1	1	1
	6	10	5	4	3	2	1	1	1
0.95	1	13	7	5	4	2	2	2	1
	2	13	7	5	4	2	2	1	1
	3	13	7	5	4	2	2	1	1
	4	13	7	5	4	2	2	1	1
	5	13	7	5	4	2	2	1	1
	6	13	7	5	4	2	2	1	1
0.99	1	19	11	7	5	3	2	2	2
	2	19	10	7	5	3	2	2	2
	3	19	10	7	5	3	2	2	2
	4	19	10	7	5	3	2	2	2
	5	19	10	7	5	3	2	2	2
	6	19	10	7	5	3	2	2	2

Table 2:

Operating Characteristic values for the time truncated chain sampling plan (n, i, T/λ_m^0) for a given p*, when i = 2 and $\alpha_0 = 2$.

p*	n	T/λ_m^0	λ/λ_0					
			2	4	6	8	10	12
0.75	6	0.628	0.757076	0.967965	0.992245	0.997317	0.99885	0.999426
	4	0.942	.650154	0.942506	0.984971	0.994605	0.99763	0.998805
	3	1.257	0.570322	0.916574	0.976721	0.991365	0.996131	0.998025
	2	1.571	0.597893	0.920967	0.977490	0.991527	0.996134	0.998027
	1	2.356	0.651007	0.929267	0.978929	0.991806	0.996203	0.998013
	1	3.141	0.459759	0.850767	0.949403	0.978936	0.989825	0.994523

	1	3.927	0.311381	0.753952	0.905844	0.958122	0.978924	0.988337
	1	4.712	0.207339	0.651007	0.850729	0.929267	0.962947	0.978929
0.90	10	0.628	0.552089	0.921562	0.979580	0.992733	0.996831	0.998412
	6	0.942	0.465283	0.886135	0.967970	0.988159	0.994721	0.997317
	4	1.257	0.432960	0.866119	0.960350	0.98493	0.993165	0.996486
	3	1.571	0.402390	0.845447	0.952020	0.98128	0.991370	0.995514
	2	2.356	0.308202	0.774366	0.921020	0.967053	0.984166	0.991534
	1	3.141	0.459759	0.850767	0.949400	0.978936	0.989825	0.994523
	1	3.927	0.311381	0.753952	0.905840	0.958122	0.978924	0.988337
	1	4.712	0.207339	0.651007	0.850730	0.929267	0.962947	0.978929
0.95	13	0.628	0.429077	0.879756	0.967004	0.988010	0.994720	0.997340
	7	0.942	0.392002	0.854898	0.957704	0.984140	0.992878	0.996370
	5	1.257	0.327943	0.811931	0.941030	0.977040	0.989462	0.994540
	4	1.571	0.270470	0.764012	0.920560	0.967910	0.984938	0.992090
	2	2.356	0.308202	0.774366	0.921020	0.967050	0.984166	0.991530
	2	3.141	0.147879	0.598114	0.829340	0.921040	0.959620	0.977510
	1	3.927	0.311381	0.753952	0.905840	0.958120	0.978924	0.988340
	1	4.712	0.207339	0.651007	0.850730	0.929270	0.962947	0.978930
0.99	19	0.628	0.258966	0.788142	0.935790	0.975710	0.989087	0.994440
	10	0.942	0.235645	0.756771	0.921560	0.969320	0.985930	0.992730
	7	1.257	0.190501	0.701951	0.895990	0.957590	0.980068	0.989540
	5	1.571	0.183723	0.68343	0.885040	0.951960	0.977060	0.987830
	3	2.356	0.149212	0.617201	0.845530	0.931160	0.965730	0.981290
	2	3.141	0.147879	0.598114	0.829340	0.921040	0.959620	0.977510
	2	3.927	0.069928	0.436299	0.716170	0.854486	0.921000	0.954140
	2	4.712	0.032935	0.308202	0.598040	0.774370	0.869480	0.921020

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