An Application of Linguistic Variables in Assignment Problem with Fuzzy Costs

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Abstract

This paper presents an assignment problem with fuzzy costs, where the objective is to minimize the cost. Here each fuzzy cost is assumed as triangular or trapezoidal fuzzy number. Yager's ranking method has been used for ranking the fuzzy numbers. The fuzzy assignment problem has been transformed into a crisp one, using linguistic variables and solved by Hungarian technique. The use of linguistic variables helps to convert qualitative data into quantitative data which will be effective in dealing with fuzzy assignment problems of qualitative nature.
A numerical example is provided to demonstrate the proposed approach.

Key words: Fuzzy Assignment Problem, Fuzzy Numbers, Hungarian method, Ranking of Fuzzy numbers

Introduction

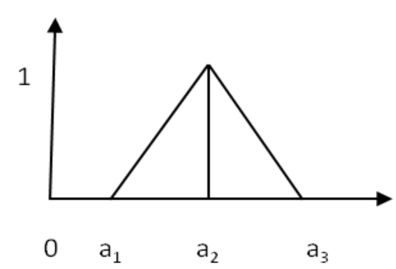
Much information that we need to deal with day to day life is vague, ambiguous, incomplete, and imprecise. Crisp logic or conventional logic theory is inadequate for dealing with such imprecision, uncertainty and complexity of the real world. It is this realization that motivated the evolution of fuzzy logic and fuzzy theory.
The fundamental concept of fuzzy theory is that any field X and theory Y can be fuzzified by replacing the concept of a crisp set in X and Y by that of a fuzzy set. Mathematically a fuzzy set [4] can be defined by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the fuzzy set. The membership function denoted by **μ** is defined from X to [0, 1].
An assignment problem (AP) is a particular type of transportation problem where n tasks (jobs) are to be assigned to an equal number of n machines (workers) in one to one basis such that the assignment cost (or profit) is minimum (or maximum). Hence, it can be considered as a balanced transportation problem in which all supplies and demands are equal, and the number of rows and columns in the matrix are identical.
Sakthi *et al* [1] adopted Yager's ranking method [2] to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve the AP. In this paper we investigate an assignment problem with fuzzy costs or times $\widetilde{C}_{ij}$ represented by linguistic variables which are replaced by triangular or trapezoidal fuzzy numbers.

Definitions and Formulations

Triangular fuzzy number

A triangular fuzzy number $\hat{a}$ is defined by a triplet (a_1, a_2, a_3). The membership function is defined as

$$\mu_{\hat{a}}(x) = \begin{cases} (x - a_1) / (a_2 - a_1) & \text{if } a_1 \le x \le a_2 \\ (a_3 - x) / (a_3 - a_2) & \text{if } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

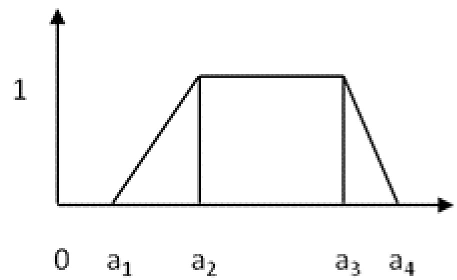


The triangular fuzzy number is based on three-value judgement: The minimum possible value a_1, the most possible value a_2 and the maximum possible value a_3

Trapezoidal fuzzy number

A trapezoidal fuzzy number $\hat{a}$ is a fuzzy number (a_1, a_2, a_3, a_4) and its membership function is defined as

$$\mu_{\hat{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ (x - a_4)/(a_3 - a_4) & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$



Linguistic Variable

A linguistic variable [3] is a variable whose values are linguistic terms. The concept of linguistic variable is applied in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions.

For example, '**height**; is a linguistic variable, its values can be very high, high, medium, low, very low etc., These values can also be represented by fuzzy numbers.

α-cut and strong α-cut

Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, the α- cut α_A, and the strong α-cut α_A+, are the crisp sets

$\alpha_A = \{x/A(x) \ge \alpha\}$

$\alpha_A+ = \{x/A(x) > \alpha\}$

The Proposed Method

The assignment problem can be stated in the form of n x n cost matrix $[c_{ij}]$ of real numbers as given in the following table:

Jobs → Persons ↓	1	2	3	---j---	n
1	$\mathbf{c_{11}}$	$\mathbf{c_{12}}$	$\mathbf{c_{13}}$	**--$\mathbf{c_{1j}}$--**	$\mathbf{c_{1n}}$
2	c_{21}	c_{22}	c_{23}	--c_{2j}--	c_{2n}
-	-	-	-	-	-
-	-	-	-	-	-
i	c_{i1}	c_{i2}	c_{i3}	--c_{ij}—	c_{in}
-	-	-	-	-	-
n	c_{n1}	c_{n2}	c_{n3}	c_{nj}	c_{nn}

Mathematically assignment problem can be stated as

Minimize $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$ i=1,2,……n: j=1,2,…….n

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,\ldots\ldots n \qquad \ldots.(1)$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j=1,2,\ldots\ldots n \qquad x_{ij} \in \{0,1\},$$

where $x_{ij} =$ 1, if the i^{th} person is assigned the j^{th} job
0, otherwise
is the decision variable denoting the assignment of the person i to job j, C_{ij} is the cost of assigning the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons. (One job to one person). When the costs $\tilde{C}_{ij}$ are fuzzy numbers, then the fuzzy assignment problem becomes

$$Y(\tilde{z}) = \sum_{i=1}^{n}\sum_{j=1}^{n} Y(\tilde{c}_{ij})x_{ij} \qquad \ldots\ldots(2)$$

subject to the same conditions (1).
We defuzzify the fuzzy cost coefficients into crisp ones by a fuzzy number ranking method. Yager's Ranking index [2] is defined by

$Y(\tilde{c}) = \int_0^1 0.5(c_\alpha^L + c_\alpha^U)$, where (c_α^L, c_α^U) is the α - level cut of the fuzzy number $\tilde{c}$.

The Yager's ranking index $Y(\tilde{c})$ gives the representative value of the fuzzy number $\tilde{c}$. Since $Y(\tilde{C}_{ij})$ are crisp values, this problem is obviously the crisp assignment problem of the form (1) which can be solved by Hungarian Method.
The steps of the proposed method are
Step 1: Replace the cost matrix C_{ij} with linguistic variables by triangular or trapezoidal fuzzy numbers.
Step 2: Find Yager's Ranking index.
Step 3: Replace Triangular or Trapezoidal numbers by their respective ranking indices.
Step 4: Solve the resulting AP using Hungarian technique to find optimal assignment.

Numerical Example
Let us consider a Fuzzy Assignment Problem with rows representing four persons W, X, Y, Z and columns representing the four jobs, Job1, Job2, Job3 and Job4 with assignment cost varying between 0$ to 50$. The cost matrix [$\tilde{C}_{ij}$] is given whose elements are linguistic variables which are replaced by fuzzy numbers. The problem is then solved by Hungarian method to find the optimal assignment.

$$\begin{array}{cc} & \begin{array}{cccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \end{array} \\ \begin{array}{c} \mathbf{W} \\ \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{array} & \begin{pmatrix} extremelylow & low & fairlyhigh & extremelyhigh \\ low & verylow & high & veryhigh \\ medium & extremelyhigh & verylow & extremelylow \\ veryhigh & low & fairlylow & fairlylow \end{pmatrix} \end{array}$$

Solution: The Linguistic variables showing the qualitative data is converted into quantitative data using the following table. As the assignment cost varies between 0$ to 50$ the minimum possible value is taken as 0 and the maximum possible value is taken as 50.

Extremely low	(0,2,5)
Very low	(1,2,4)
Low	(4,8,12)
Fairly low	(15,18,20)
Medium	(23,25,27)
Fairly High	(28,30,32)
High	(33,36,38)
Very High	(37,40,42)
Extremely High	(44,48,50)

The linguistic variables are represented by triangular fuzzy numbers
Now

$$\begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\
\begin{pmatrix}
(0,2,5) & (4,8,12) & (28,30,32) & (44,48,50) \\
(4,8,12) & (1,2,4) & (33,36,38) & (37,40,42) \\
(23,25,27) & (44,48,50) & (1,2,4) & (0,2,5) \\
(37,40,42) & (4,8,12) & (15,18,20) & (15,18,20)
\end{pmatrix}
\end{array} \qquad \ldots(3)$$

we calculate Y(0,2,5) by applying the Yager's Ranking Method.
The membership function of the triangular fuzzy number (0,2,5) is

$$\mu(x) = \begin{array}{l} \dfrac{x-0}{2-0}, 0 \le x \le 2 \\ \dfrac{x-5}{2-5} 2 \le x \le 5 \end{array}$$

The α − cut of the fuzzy number (0,2,5) is $(c_\alpha{}^L, c_\alpha{}^U) = (2\alpha, 5-3\alpha)$ for which

$$Y(\tilde{c}_{11}) = Y(0,2,5) = \int_0^1 0.5(c_\alpha^L, c_\alpha^U)d\alpha = \int_0^1 0.5(2\alpha + 5 - 3\alpha)d\alpha = 2.25$$

Proceeding similarly, the Yager's indices for the costs $\tilde{c}_{ij}$ are calculated as:

$Y(\tilde{c}_{12}) = 8$, $Y(\tilde{c}_{13}) = 31$, $Y(\tilde{c}_{14}) = 47.5$, $Y(\tilde{c}_{21}) = 8$, $Y(\tilde{c}_{22}) = 1.75$, $Y(\tilde{c}_{23}) = 81.5$, $Y(\tilde{c}_{24}) = 79.5$, $Y(\tilde{c}_{31}) = 25$,

$Y(\tilde{c}_{32}) = 47.5$, $Y(\tilde{c}_{33}) = 1.75$, $Y(\tilde{c}_{34}) = 2.25$, $Y(\tilde{c}_{41}) = 79.5$, $Y(\tilde{c}_{42}) = 8$, $Y(\tilde{c}_{43}) = 35.5$, $Y(\tilde{c}_{44}) = 35.5$.

We replace these values for their corresponding $\tilde{c}_{ij}$ in (3) and solve the resulting assignment problem by using Hungarian method.

$$\begin{pmatrix}
2.25 & 8 & 31 & 47.5 \\
8 & 1.75 & 35.75 & 39.75 \\
25 & 47.5 & 1.75 & 2.25 \\
39.75 & 8 & 17.75 & 17.75
\end{pmatrix}$$

Performing row reductions

$$\begin{pmatrix}
0 & 5.75 & 28.75 & 45.25 \\
6.25 & 0 & 34 & 38 \\
23.25 & 45.75 & 0 & 0.5 \\
31.75 & 0 & 9.75 & 9.75
\end{pmatrix}$$

Performing column reductions

$$\begin{pmatrix}
0 & 5.75 & 28.75 & 44.75 \\
6.25 & 0 & 34 & 37.5 \\
23.25 & 45.75 & 0 & 0 \\
31.75 & 0 & 9.75 & 9.25
\end{pmatrix}$$

The optimal assignment matrix is

$$\begin{pmatrix} 0 & 5.75 & 19.5 & 44.75 \\ 6.25 & 0 & 24.75 & 28.25 \\ 32.5 & 55 & 0 & 0 \\ 31.75 & 0 & 0.5 & 0 \end{pmatrix}$$

The optimal assignment schedule is $W \to 1, X \to 2, Y \to 3, Z \to 4$.

Conclusions:

In this paper, the assignment costs are considered as linguistic variables represented by fuzzy numbers. The fuzzy assignment problem has been transformed into crisp assignment problem using Yager's ranking indices. Hence we have shown that the fuzzy assignment problems of qualitative nature can be solved in an effective way. This technique can also be tried in solving other types of problems like Transportation problems, project scheduling problems, network flow problems etc.,

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