To Find Strong Dominating Set and Split Strong Dominating Set of an Interval Graph Using an Algorithm

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Abstract:
Strong and weak domination arise naturally in certain practical situations. For example, consider a network of roads connecting a number of locations. In such a network, the degree of a vertex v is the number of roads meeting at v. Suppose $\deg u \geq \deg v$. Naturally, the traffic at u is heavier than that at v. If we consider the traffic between u and v, preference should be given to the vehicles going from u to v. Thus, in some sense, u strongly dominates v and v weakly dominates u. In this paper we present an algorithm to find a strong dominating set and split strong dominating set of an interval graph which is connected.

Keywords: domination number, dominating set, Interval family, Interval graph, strong domination number , split dominating set, spit strong dominating set, strong dominating set.

1. Introduction

We have defined a graph as a set and a certain relation on that set. It is often convenient to draw a "picture" of the graph. This may be done in many ways usually one draws an interval graph corresponding to I for each vertex and connects vertex u and vertex v with a directed arrow whenever uv is an edge. If both uv and vu are edges then some times a single line joints u and v without arrows.

Let $I = \{I_1, I_2, ..., I_n\}$ be the given interval family. Each interval i in I is represented by $[a_i, b_i]$, for $i = 1, 2, ..., n$. Here a_i is called the left endpoint and b_i the right endpoint of the interval I_i. Without loss of generality we may assume that all end points of the intervals in I which are distinct between 1and 2n. The intervals are labelled in the increasing order of their right endpoints. Two intervals i and j are said to intersect each other, if they have non-empty intersection. Interval graphs play important role in numerous applications, many of which are scheduling problems. They are a subset of perfect graphs [1]. A graph $G = (V, E)$ is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. That is, if $i = [a_i, b_i]$ and $j = [a_j, b_j]$, then i and j intersect means either $a_j < b_i$ or $a_i < b_j$.

Let G be a graph, with vertex set V and edge set E. The open neighbourhood set of a vertex $v \in V$ is

$$nbd(v) = \{u \in V / uv \in E\}$$

The closed neighbourhood set of a vertex $v \in V$ is

$$nbd[v] = nbd(v) \cup \{v\}$$

A vertex in a graph G dominates itself and it's neighbours. A set $D \subseteq V$ is called dominating set if every vertex in $< V - D >$ is adjacent to some vertex in D. The domination studied in [2-3]. The domination number γ of G is the minimum cardinality of a dominating set. The domination number is well-studied parameter. We can see this from the bibliography [4] on domination. In [5] , Sampathkumar and Pushpa Latha have introduced the concept of strong domination in graphs. Strong domination has been studied [6-67]. Kulli.V.R. et all [8] introduced the concept of split and non-split domination in graphs. A dominating set D is called split dominating set if the induced subgraph $< V - D >$ is disconnected. The split domination number of γ_s of G is the minimum cardinality of a split dominating set. Let $G = (V, E)$ be a graph and $u, v \in V$. Then u strongly dominates v if

(i) $uv \in E$
(ii) $\deg v \leq \deg u$.

A set $D_{st} \subseteq V$ is a strong dominating set of G if every vertex in $V - D_{st}$ is strongly dominated by atleast one vertex in D_{st}. The strong domination number $\gamma_{st}(G)$ of G is the minimum cardinality of a strong dominating set. Define

$NI(i) = j$, if $b_i < a_j$ and there do not exist an interval k such that $b_i < a_k < a_j$. If there is no such j, then define $NI(i) = null$. N_{sd} (i) is the set of all neighbours whose degree is greater than degree of i and also greater than i. If there is no such neighbor then define $N_{sd}(i) = null$. M (S) is the largest highest degree vertex in the set S. nbd^+ (i) is the set of all adjacent vertices which are greater than i. nbd^- (i) is the set of all adjacent vertices which are less than i. d^+ (i) is the number of adjacent vertices which are greater than i. d^- (i) is the number of adjacent vertices which are less than i.

3. Algorithms

3.1. To find a Strong dominating set of an interval graph using an algorithm

Input: Interval family $I = \{I_1, I_2, ..., I_n\}$.

Output: Strong dominating set of an interval graph of a given interval family.

Step 1 : $S_1 = nbd$ [1].

Step 2 : S = The set of vertices in S_1 which are adjacent to all other vertices in S_1.

Step 3 : D_{st} = The largest highest degree interval in S.

Step 4 : LI = The largest interval in D_{st}.

Step 5 : If $N_{sd}(LI)$ exists

Step 5.1 : a = $M(N_{sd}(LI))$.

Step 5.2 : b = The largest highest degree interval in nbd [a].

Step 5.3 : $D_{st} = D_{st} \cup \{b\}$ goto step 4.

end if

else

Step 6 : Find $NI(LI)$

Step 6.1: If $NI(LI) = null$ goto step 7.

Step 6.2 : $S_2 = nbd[NI(LI)]$.

Step 6.3 : S_3 = The set of all neighbors of $NI(LI)$ which are greater than or equal to $NI(LI)$.

Step 6.4 : S_4 = The set of all vertices in S_3 which are adjacent to all vertices in S_3.

Step 6.5 : c = The largest highest degree interval in S_4.

Step 6.6 : $D_{st} = D_{st} \cup \{c\}$ goto step 4.

Step 7 : End

3.2. To find a split strong dominating set(sd_{st}) of an interval graph using an algorithm.

Input : Interval family $I = \{I_1, I_2, ..., I_n\}$.

Output : Split strong dominating set of an interval graph of an interval family I.

Step 1 : $S_1 = nbd$ [1].

Step 2 : S_2 =The set of all vertices in S_1 which are adjacent to all other vertices in S_1.

Step 3 : $SD_{st} = \{a\}$, where ‘a’ is the largest highest degree interval in S_2.

Step 4 : Count = The number of pendent vertices or number of vertices with degree one in G.

Step 5 : If Count > 0 then goto step 7

Else

Step 6 : If there exists at least one edge (u, v) such that $u \in nbd^-(a)$ & $v \in nbd^+(a)$

Step 6.1 : Count = Count + 1.

Step 6.2 : Take largest v.

Step 6.3 : $SD_{st} = SD_{st} \cup \{v\} = \{a, v\}$.

Endif

Step 7 : LI = The largest interval in SD_{st}.

Step 8 : If $d^+(LI)=1$ then (If already checked this condition for the same vertex skip)

Step 8.1 : Count = Count + 1.

Step 9 : If $Count \geq 1$ then goto step 12

Else

Step 10 : If there exists atleast one edge (w,x) such that $w \in nbd^-(LI)$ & $x \in nbd^+(LI)$

Step 10.1 : Count = Count + 1.

Step 10.2 : Take largest x.

Step 10.3 : $SD_{st} = SD_{st} \cup \{x\}$.

Endif

Step 11 : If $d^+(x)=1$

Step 10.1 : Count = Count + 1.

Step 12 : If $N_{sd}(LI)$ exists

Step 12.1 : a = $M(N_{sd}(LI))$.

Step 12.2 : b = The largest highest degree interval in nbd [a].

Step 12.3 : $SD_{st} = SD_{st} \cup \{b\}$ goto step 7.

end if

else

Step 13 : Find $NI(LI)$

Step 13.1: If $NI(LI) = null$ goto step 14.

Step 13.2 : $S_2 = nbd[NI(LI)]$.

Step 13.3 : S_3 = The set of all neighbors of $NI(LI)$ which are greater than or equal to $NI(LI)$.

Step 13.4 : S_4 = The set of all vertices in S_3 which are adjacent to all vertices in S_3.

Step 13.5 : c = The largest highest degree interval in S_4.

Step 13.6 : $D_{st} = D_{st} \cup \{c\}$ goto step 7.

Step 14 : End

4. Main Theorems.

Theorem 4.1 : Let G be an interval graph corresponding to an interval family $I=\{I_1, I_2, ..., I_n\}$. If i and j are any two intervals in I such that $i \in D_{st}$, where D_{st} is a minimum strong dominating set of the given interval graph G, $j \neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and at least one interval $k \neq i$ to the right of j that intersects j then $\gamma_{st}(G) < \gamma_{sst}(G)$.

Proof : Let G be an interval graph corresponding to an interval family $I=\{I_1, I_2, ..., I_n\}$. Let i and j be any two intervals in I such that $i \in D_{st}$, where D_{st} is a minimum strong dominating set of the given interval graph G, $j \neq 1$ and j is contained in i and Suppose there is at least one interval to the left of j that intersects j and at least one interval $k \neq i$ to the right of j that intersects j. Then it is obviously we know that j is adjacent to k in the induced sub graph $<V - D_{st}>$. Then there will be a connection in $<V - D_{st}>$. Since there is at least one interval to the left of j that intersects j, there will be a connection in $<V - D_{st}>$ to it's left. In this connection we introduce another interval 'h', which is to the right of j and i and also interest i and j to D_{st} for disconnection in the induced subgraph $<V - D_{st}>$. We also formulated Split strong dominating set as follows

$$SD_{st} = D_{st} \cup \{h\} \Rightarrow |SD_{st}| = |D_{st} \cup \{h\}|.$$

Since D_{st}, h are disjoint $\Rightarrow |SD_{st}| = |D_{st}| + |\{h\}|$ Or $|D_{st}| + |\{h\}| = |SD_{st}|$.

$$\Rightarrow \gamma_{st}(G) + |\{h\}| = \gamma_{sst}(G).$$

$$\Rightarrow \gamma_{st}(G) < \gamma_{sst}(G).$$

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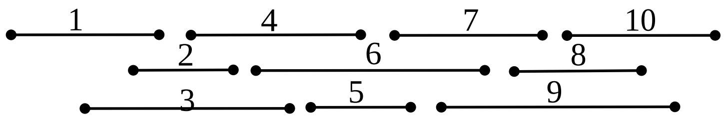

Figure 1. Interval Family

As follows an algorithm with illustration for neighbours as given interval family I.
We construct an interval graph G from an interval family I={1, 2,……,10}as follows

$\text{nbd}\,[1]=\{1,2,3\}$, $\text{nbd}\,[2]=\{1,2,3,4\}$, $\text{nbd}\,[3]=\{1,2,3,4,6\}$, $\text{nbd}\,[4]=\{2,3,4,5,6\}$, $\text{nbd}\,[5]=\{4,5,6,7\}$, $\text{nbd}\,[6]=\{3,4,5,6,7,9\}$, $\text{nbd}\,[7]=\{5,6,7,8,9\}$, $\text{nbd}\,[8]=\{7,8,9,10\}$, $\text{nbd}\,[9]=\{6,7,8,9,10\}$, $\text{nbd}\,[10]=\{8,9,10\}$.

$N_{sd}(1)=\{2,3\}$, $N_{sd}(2)=\{3,4\}$, $N_{sd}(3)=\{6\}$, $N_{sd}(4)=\{6\}$, $N_{sd}(5)=\{6\}$, $N_{sd}(6)=\text{null}$, $N_{sd}(7)=\text{null}$, $N_{sd}(8)=\{9\}$, $N_{sd}(9)=\text{null}$, $N_{sd}(10)=\text{null}$.

$\text{NI}\,(1)=4$, $\text{NI}\,(2)=5$, $\text{NI}\,(3)=5$, $\text{NI}\,(4)=7$, $\text{NI}\,(5)=8$, $\text{NI}\,(6)=8$, $\text{NI}\,(7)=10$, $\text{NI}\,(8)=\text{null}$, $\text{NI}\,(9)=\text{null}$, $\text{NI}\,(10)=\text{null}$.

Procedure for finding a strong dominating set(D_{st}) of an interval graph using an algorithm.

Input : Interval family $I=\{1,2,......,10\}$.

Step 1 **:** $S_1=\{1,2,3\}$.

Step 2 : $S=\{1,2,3\}$.

Step 3 : $D_{st}=\{3\}$.

Step 4 : $LI=3$.

Step 5 : $N_{sd}(3)=\{6\}$.

Step 5.1 : $a=M(N_{sd}(3))=M(\{6\})=6$.

Step 5.2 : $b=6$.

Step 5.3 : $D_{st}=\{3\}\cup\{6\}=\{3,6\}$.

Step 6 : $LI=6$.

Step 7 : $NI\ (6)=8$.

Step 7.1 : $S_2=nbd\ [8]=\{7,8,9,10\}$.

Step 7.2 : $S_3=\{8,9,10\}$.

Step 7.3 : $S_4=\{8,9,10\}$.

Step 7.4 : $c=9$.

Step 7.5 : $D_{st}=D_{st}\cup\{9\}=\{3,6\}\cup\{9\}=\{3,6,9\}$.

Step 8 : $LI=9$.

Step 9 : $N_{sd}(9)$ = null and NI (9) = null.

Step 10 : End.

Output : {3,6,9} is the strong dominating set of an interval graph of given interval family.

Procedure for finding a split strong dominating set(SD_{st}) of an interval graph using an algorithm.

Input : Interval family $I=\{1,2,......,10\}$.

Step 1 **:** $S_1=\{1,2,3\}$.

Step 2 : $S_2=\{1,2,3\}$.

Step 3 : $SD_{st}=\{3\}$.

Step 4 : Count = 0.

Step 5 : There exists (2,4) such that $2\in nbd^-(3)=\{1,2\}$ and $4\in nbd^+(3)=\{4,6\}$.

Step 5.1 : Count = Count +1 .
Step 5.2 : $SD_{st} = \{3\} \cup \{4\} = \{3,4\}$.
Step 6 : $LI = 4$.
Step 7 : $N_{sd}(4) = \{6\}$.
Step 5.1 : $a = M(N_{sd}(4)) = M(\{6\}) = 6$.
Step 5.2 : $b = 6$.
Step 5.3 : $SD_{st} = \{3,4\} \cup \{6\} = \{3,4,6\}$.
Step 8 : $LI = 6$.
Step 9 : $NI\ (6) = 8$.
Step 9.1 : $S_2 = nbd\ [8] = \{7,8,9,10\}$.
Step 9.2 : $S_3 = \{8,9,10\}$.
Step 9.3 : $S_4 = \{8,9,10\}$.
Step 9.4 : $c = 9$.
Step 9.5 : $D_{st} = D_{st} \cup \{9\} = \{3,6\} \cup \{9\} = \{3,6,9\}$.
Step 10 : $LI = 9$.
Step 11 : $N_{sd}(9)$ = null and NI (9) = null.
Step 12 : End.

Output : {3,4,6,9} is the split strong dominating set of an interval graph of given interval family.

$D_{st} = \{3,6,9\}$.

$SD_{st} = \{3,4,6,9\}$.

$|D_{st}| < |SD_{st}|$.

$\therefore \gamma_{st}(G) < \gamma_{sst}(G)$.

Theorem 4.2 : Let D_{st} be a strong dominating set of the given interval graph G corresponding to an interval family $I = \{I_1, I_2, ..., I_n\}$. If i and j are any two intervals in I such that j is contained in i and if there is no other interval $k \neq i$ that intersects j then the strong dominating set D_{st} is also a split strong dominating set of an interval graph G.

Proof : Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family and G is an interval graph corresponding to I . Let i and j be any two intervals in I such that j is contained in i . If there is no interval $k \neq i$ that intersect j . Then clearly i lies in the strong dominating set D_{st} . Further in induced subgraph $< V - D_{st} >$ the vertex j is not adjacent to any other vertex and then j becomes as an solated vertex in induced sub graph $< V - D_{st} >$. There is a disconnection in $< V - D_{st} >$ .Hence the strong dominating set, which we considered is split strong dominating set.Hence we follows an algorithm to find strong dominating set and split strong dominating set of an interval graph with an illustration.

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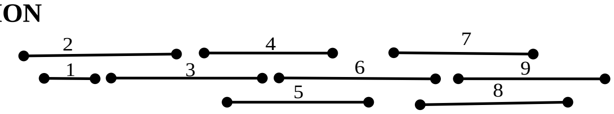

Figure 2.Interval Family

As follows an algorithm with illustration for neighbours as given interval family I.
We construct an interval graph G from an interval family I={1, 2,……,9}as follows

$\text{nbd}[1] = \{1,2\}$, $\text{nbd}[2] = \{1, 2, 3\}$, $\text{nbd}[3] = \{2,3,4,5\}$, $\text{nbd}[4] = \{3,4,5,6\}$, $\text{nbd}[5] = \{3,4,5,6\}$,
$\text{nbd}[6] = \{4,5,6,7,8\}$, $\text{nbd}[7] = \{6,7,8,9\}$, $\text{nbd}[8] = \{6,7,8,9\}$, $\text{nbd}[9] = \{7,8,9\}$
$N_{sd}(1) = \{2\}$, $N_{sd}(2) = \{3\}$, $N_{sd}(3) = \text{null}$, $N_{sd}(4) = \{6\}$, $N_{sd}(5) = \{6\}$, $N_{sd}(6) = \text{null}$,
$N_{sd}(7) = \text{null}$, $N_{sd}(8) = \text{null}$, $N_{sd}(9) = \text{null}$.
$NI\ (1) = 3$, $NI\ (2) = 4$, $NI\ (3) = 6$, $NI\ (4) = 7$, $NI\ (5) = 7$, $NI\ (6) = 9$, $NI\ (7) = \text{null}$,

$\mathrm{NI}\,(8) = \text{null}$, $\mathrm{NI}\,(9) = \text{null}$.

Procedure for finding a strong dominating set(D_{st}) of an interval graph using an algorithm.

Input : Interval family $I = \{1, 2, ..., 9\}$.

Step 1 **:** $S_1 = nbd[1] = \{1, 2\}$.

Step 2 : $S = \{1, 2\}$.

Step 3 : $D_{st} = \{2\}$.

Step 4 : $LI = 2$.

Step 5 : $N_{sd}(2) = \{3\}$.

Step 5.1 : $a = M(N_{sd}(2)) = M(\{3\}) = 3$.

Step 5.2 : $b = 5$.

Step 5.3 : $D_{st} = \{2\} \cup \{5\} = \{2, 5\}$.

Step 6 : $LI = 5$.

Step 7 : $N_{sd}(5) = \{6\}$.

Step 7.1 : $a = M(N_{sd}(5)) = M(\{6\}) = 6$.

Step 7.2 : $b = 6$.

Step 7.3 : $D_{st} = \{2, 5\} \cup \{6\} = \{2, 5, 6\}$.

Step 8 : $LI = 6$.

Step 9 : $NI(6) = 9$.

Step 9.1 : $S_2 = nbd\ [9] = \{7,8,9\}$.

Step 9.2 : $S_3 = \{9\}$.

Step 9.3 : $S_4 = \{9\}$.

Step 9.4 : $c = 9$.

Step 9.5 : $D_{st} = D_{st} \cup \{9\} = \{2, 5, 6\} \cup \{9\} = \{2, 5, 6, 9\}$.

Step 10 : $LI = 9$.

Step 11: $N_{sd}(9)$ = null and NI (9) = null.

Step 12 : End.

Output : {2,5,6,9} is the strong dominating set of an interval graph of given interval family.

Procedure for finding a split strong dominating set(SD_{st}) of an interval graph using an algorithm.

Input : Interval family $I = \{1, 2, ..., 9\}$.

Step 1: $S_1 = nbd[1] = \{1, 2\}$.

Step 2 : $S = \{1, 2\}$.

Step 3 : $SD_{st} = \{2\}$.

Step 4 : Count = 1.

Step 5 : $LI = 2$.

Step 6 : $d^+(2) = 1$.

Step 6.1 : Count = 1 + 1 = 2.

Step 7 : $N_{sd}(2) = \{3\}$.

Step 7.1 : $a = M(N_{sd}(2)) = M(\{3\}) = 3$.

Step 7.2 : $b = 5$.

Step 7.3 : $D_{st} = \{2\} \cup \{5\} = \{2, 5\}$.

Step 8 : $LI = 5$.

Step 9 : $N_{sd}(5) = \{6\}$.

Step 9.1 : $a = M(N_{sd}(4)) = M(\{6\}) = 6$.

Step 9.2 : $b = 6$.

Step 9.3 : $SD_{st} = \{2,5\} \cup \{6\} = \{2,5,6\}$.
Step 10 : $LI = 6$.
Step 11 : $NI\ (6) = 9$.
Step 11.1 : $S_2 = nbd\ [9] = \{7,8,9\}$.
Step 11.2 : $S_3 = \{9\}$.
Step 11.3 : $S_4 = \{9\}$.
Step 11.4 : $c = 9$.
Step 11.5 : $D_{st} = D_{st} \cup \{9\} = \{2,5,6\} \cup \{9\} = \{2,5,6,9\}$.
Step 12 : $LI = 9$.
Step 13 : $N_{sd}(9)$ = null and NI (9) = null.
Step 14 : End.

Output : {2,5,6,9} is the split strong dominating set of an interval graph of given interval family.
$D_{st} = \{2,5,6,9\}$, $SD_{st} = \{2,5,6,9\}$.
$D_{st} = SD_{st}$.

Theorem 4.3 : Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family and D_{st} is a strong dominating set of the given interval graph G. If i, j, k are any three consecutive intervals such that $i < j < k$ and if $j \in D_{st}$, and i intersects j, j intersect k and i does not intersect k then $D_{st} = SD_{st}$.

Proof : Suppose $I = \{I_1, I_2, ..., I_n\}$ be an interval family . If i, j, k be three consecutive intervals such that $i < j < k$ and i intersect j, j intersect k, but i does not intersect k. Suppose $j \in D_{st}$, where D_{st} is a strong dominating set. Then i and k are not adjacent in the induced subgraph $< V - D_{st} >$. There exists a a disconnection between i and k. That is, there is no $m \in I$, $m < k$ such that m intersects k. If possible suppose that such an m exists, then since $m < k$ we must have $m < i < j < k$ ($\because m < k$). Now m intersects k implies i and j also intersect. Then there is a path between i and k and are adjacent. This is a contradiction to hypothesis. So such a m does not exists. Hence we get disconnection. Hence D_{st} is also a split strong dominating set of the given interval graph G. As usual as follows an algorithm to find a strong dominating set and split strong dominating set of an interval graph G.

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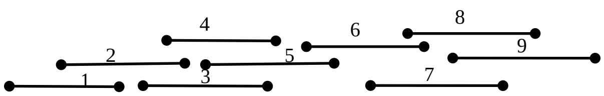

Figure 3.Interval Family I.

We construct an interval graph from an interval family I={1,2,3,4,5,6,7,8,9} as follows.

$\text{nbd}[1] = \{1,2\}$, $\text{nbd}[2] = \{1,2,3,4\}$, $\text{nbd}[3] = \{2,3,4,5\}$, $\text{nbd}[4] = \{2,3,4,5\}$, $\text{nbd}[5] = \{3,4,5,6\}$,
$\text{nbd}\ [6] = \{5,6,7,8\}$, $\text{nbd}\ [7] = \{6,7,8,9\}$, $\text{nbd}[8] = \{6,7,8,9\}$, $\text{nbd}[9] = \{7,8,9\}$.
$N_{sd}(1) = \{2\}$, $N_{sd}(2) = \text{null}$, $N_{sd}(3) = \text{null}$, $N_{sd}(4) = \text{null}$, $N_{sd}(5) = \text{null}$, $N_{sd}(6) = \text{null}$,
$N_{sd}(7) = \text{null}$, $N_{sd}(8) = \text{null}$, $N_{sd}(9) = \text{null}$.
$\text{NI}\ (1) = 3$, $\text{NI}\ (2) = 5$, $\text{NI}\ (3) = 6$, $\text{NI}\ (4) = 6$, $\text{NI}(5) = 7$, $\text{NI}\ (6) = 9$, $\text{NI}\ (7) = \text{null}$,
$\text{NI}\ (8) = \text{null}$, $\text{NI}\ (9) = \text{null}$.

Procedure for finding a strong dominating set(D_{st}) of an interval graph using an algorithm.

Input : Interval family $I = \{1, 2, ..., 9\}$.
Step 1 : $S_1 = nbd[1] = \{1,2\}$.
Step 2 : $S = \{1,2\}$.
Step 3 : $D_{st} = \{2\}$.
Step 4 : $LI = 2$.
Step 5 : $NI(2) = 5$.

Step 5.1 : $S_2 = nbd\ [5] = \{3,4,5,6\}$.

Step 5.2 : $S_3 = \{5,6\}$.

Step 5.3 : $S_4 = \{5,6\}$.

Step 5.4 : $c = 6$.

Step 5.5 : $D_{st} = D_{st} \cup \{6\} = \{2\} \cup \{6\} = \{2,6\}$.

Step 6 : $LI = 6$.

Step 7 : $NI(6) = 9$.

Step 7.1 : $S_2 = nbd\ [9] = \{7,8,9\}$.

Step 7.2 : $S_3 = \{9\}$.

Step 7.3 : $S_4 = \{9\}$.

Step 7.4 : $c = 9$.

Step 7.5 : $D_{st} = D_{st} \cup \{6\} = \{2,6\} \cup \{9\} = \{2,6,9\}$.

Step 8 : $LI = 9$.

Step 9: $N_{sd}(9)$ = null and NI (9) = null.

Step 12 : End.

Output : {2,6,9} is the strong dominating set of an interval graph of given interval family.

Procedure for finding a split strong dominating set(SD_{st}) of an interval graph using an algorithm.

Input : Interval family $I = \{1,2,......,9\}$.

Step 1 : $S_1 = nbd[1] = \{1,2\}$.

Step 2 : $S = \{1,2\}$.

Step 3 : $SD_{st} = \{2\}$.

Step 4 : Count = 1.

Step 5 : $LI = 2$.

Step 6 : $d^+(2) = 1$.

Step 6.1 : Count = 1 + 1 = 2.

Step 7 : $N_{sd}(2) = \{3\}$.

Step 7.1 : $a = M(N_{sd}(2)) = M(\{3\}) = 3$.

Step 7.2 : $b = 5$.

Step 7.3 : $D_{st} = \{2\} \cup \{5\} = \{2,5\}$.

Step 8 : $LI = 5$.

Step 9 : $N_{sd}(5) = \{6\}$.

Step 9.1 : $a = M(N_{sd}(4)) = M(\{6\}) = 6$.

Step 9.2 : $b = 6$.

Step 9.3 : $SD_{st} = \{2,5\} \cup \{6\} = \{2,5,6\}$.

Step 10 : $LI = 6$.

Step 11 : $NI\ (6) = 9$.

Step 11.1 : $S_2 = nbd\ [9] = \{7,8,9\}$.

Step 11.2 : $S_3 = \{9\}$.

Step 11.3 : $S_4 = \{9\}$.

Step 11.4 : $c = 9$.

Step 11.5 : $D_{st} = D_{st} \cup \{9\} = \{2,5,6\} \cup \{9\} = \{2,5,6,9\}$.

Step 12 : $LI = 9$.

Step 13 : $N_{sd}(9)$ = null and , NI (9) = null.

Step 14 : End.

Output : {2,5,6,9} is the split strong dominating set of an interval graph of given interval family.

$D_{st} = \{2,5,6,9\}$

$SD_{st} = \{2,5,6,9\}$.

$\therefore D_{st} = SD_{st}$.

5. Conclusions

In this paper we introduced an algorithm for finding strong dominating set and split strong dominating set of an interval graph which is connected.

6. Acknowledgements

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