Performance Evaluation of different α value for OFDM System

Dr. K.Elangovan
Dept. of Computer Science & Engineering
Bharathidasan University
Trichirappalli

Abstract:
Orthogonal Frequency Division Multiplexing (OFDM) has recently been applied in wireless communication systems due to its high data rate transmission capability with high bandwidth efficiency and its robustness to multi-path delay. Fading is the one of the major aspect which is considered in the receiver. In this paper the Performance Evaluation of α (0.05, 0.005 and 0.0005) values for OFDM System using LMS Algorithm. These different α value is considered in this work and their performances are statistically compared by using computer simulations.

Keywords: OFDM, BPSK and QPSK modulations and LMS Algorithm

1.Introduction

Orthogonal frequency division multiplexing (OFDM) is a modulation and multiple access technique that has been explored for over 20 years [1]. Only recently has it been finding its way into commercial communications systems, as Moore's Law has driven down the cost of the signal processing that is needed to implement OFDM based systems. OFDM, or multitone modulation, is presently used in a number of commercial wired and wireless applications. On the wired side, it is used for a variant of digital subscriber line (DSL). For wireless, OFDM is the basis for several television and radio broadcast applications, including the European digital broadcast television standard, as well as digital radio in North America. OFDM is also utilized in several fixed wireless systems and wireless local area network products.

All modern mobile wireless systems employ a variety of techniques to combat the challenges of the wireless channel[2]. Some techniques are more effective than others, with the effectiveness depending on both the air interface and system architecture approach taken to satisfy the requirements of the services being offered. As mobile systems evolved from analog to digital, more sophisticated signal processing techniques have been employed to overcome the wireless environment. These techniques include diversity, equalization, channel or error correction coding, spread spectrum, interleaving, and more recently, space time coding[3].Diversity has long been used to help mitigate the multipath induced fading that results from users' mobility. The simplest diversity technique, spatial diversity, involves the use of two or more receive antennas at a base station, separated by some distance, say on the order of five to ten wavelengths. The signal from the mobile will generally follow separate paths to each antenna. This relatively low cost approach yields significant performance improvement by taking advantage of the statistical likelihood that the paths are not highly correlated with each other. When one antenna is in a fade, the other one will generally not be [3].

A striking result is that the bit error probability of quadrature phase shift keying (QPSK) is identical to binary phase shift keying (BPSK), but twice as much data can be sent in the same bandwidth. Thus when compared to BPSK, QPSK provides twice the spectral efficiency with exactly the same energy efficiency. Similar BPSK, QPSK can also be differently encoded to allow non coherent detection[4].

Phase Shift Keying is a digital modulation scheme that conveys data by changing, or modulating, the phase of a reference carrier signal. Any digital modulation scheme uses a finite number of distinct signals to represent digital data. PSK uses a finite number of phases, each assigned a unique pattern of binary bits. Usually, each phase encodes an equal number of bits. Each pattern of bits forms the symbol that is represented by the particular phase. BPSK and QPSK are four major Modulation Techniques used in most of the applications.

2. Basic OFDM Block Diagram

The figure 2.1. shown below is the basic block diagram of the OFDM transmitter and receiver.

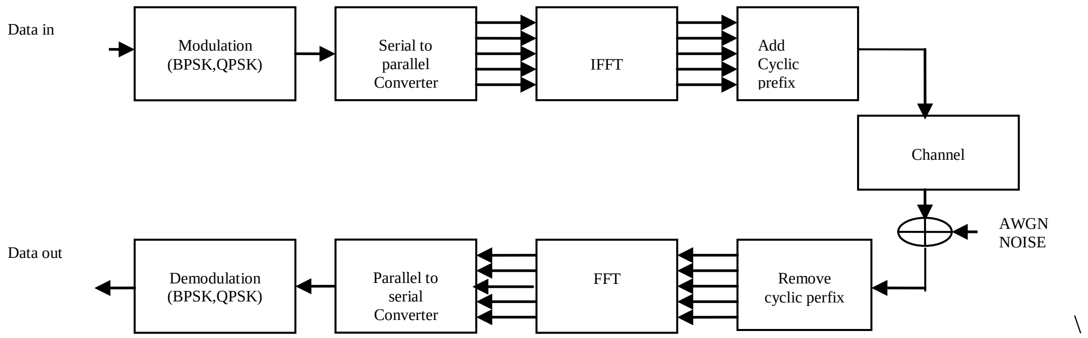

Fig. 2.1. OFDM Block Diagram.

3. Modulation Of Data

The data to be transmitted on each carrier is to be modulated using any modulation technique depending upon the user requirements. If a single bit is to be transmitted over a modulated symbol BPSK can be used. To transmit two data bits per symbol, QPSK can be made use of. In case of transmission of three bits per modulated symbol, 8- Phase Shift Keying (8-PSK) is used.

In Binary Phase Shift Keying (BPSK), the phase of a constant amplitude carrier signal is switched between two values according to the two possible signals m_1 and m_2 corresponding to binary 1 and 0, respectively. Normally, two phases are separated by 180º. If the sinusoidal carrier has an amplitude A_c and energy per bit $E_b = ½ A_c^2 T_b$, then the transmitter BPSK signal is either

$$S_{BPSK}(t) = (2E_b/T_b)^{1/2} \cos(2\Pi f_c t + \theta_c) \quad 0<t<T_b \text{ (for binary 1)} \qquad (3.1)$$

or

$$S_{BPSK}(t) = -(2E_b/T_b)^{1/2} \cos(2\Pi f_c t + \theta_c) \quad 0<t<T_b \text{ (for binary 0)} \qquad (3.2)$$

The BPSK signal is equivalent to a double sideband suppressed carrier amplitude modulated waveform, where $\cos(2\Pi f_c t)$ is applied as a carrier, and the data signal $m(t)$ is applied as the modulating waveform.

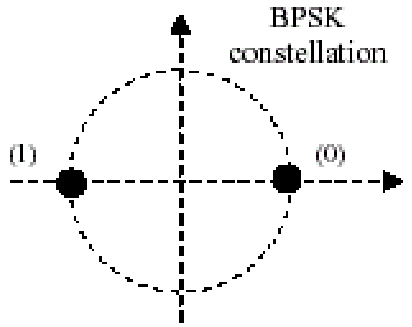

Fig. 3.1. Mapping bits into BPSK constellation.

Quadrature phase shift keying (QPSK), has twice the bandwidth efficiency of BPSK, since two bits are transmitted in a single modulation symbol. The phase of the carrier takes on one of four equally spaced values, such as 0º, 90º, 180º and 270º, where each value of phase corresponds to a unique pair of message bits. The QPSK signal for this set of symbol states may be defined as,

$$S_{QPSK}(t) = (2E_s/T_s)^{1/2} \cos(2\Pi f_c t + (i-1)\,\Pi/2) \quad 0<t<T_s \quad i=1,2,3,4 \qquad (3.3)$$

Where T_s is the symbol duration and is equal to twice the bit period.

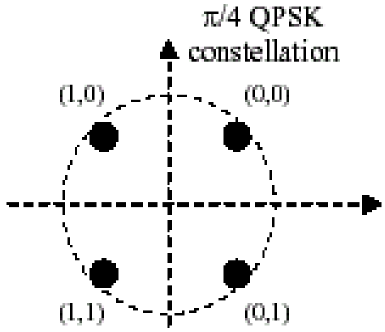

Fig. 3.2. Mapping bits into QPSK constellation.

A striking result is that the bit error probability of QPSK is identical to BPSK, but twice as much data can be sent in the same bandwidth. Thus when compared to BPSK, QPSK provides twice the spectral efficiency with exactly the same energy efficiency. Similar to BPSK, QPSK can also be differently encoded to allow non coherent detection.

4. Channel Estimation using LMS Algorithm

Channel estimation has been studied using various algorithms by an assortment of researchers[5]. In this work Channel estimation is done with the help of LMS algorithms. These techniques have shown better results than the conventional techniques. The suitability of the proposed model has been investigated and quantified. We present the results of simulations and analysis, in next chapter.

A most robust equalizer is the LMS equalizer where the criterion used is the minimization of the Mean Square Error (MSE) between the desired equalizer output and the actual equalizer output. Using the notations given below, the LMS algorithm can be readily understood.

New weights = Previous weights + (α) x (Previous error) x (Current input vector)
Where
Previous error = Previous desired output – Previous actual output

where α is constant and the constant may be adjusted by the algorithm to control the variation between the filter weights on successive iterations. The block diagram of LMS equalizer is shown.

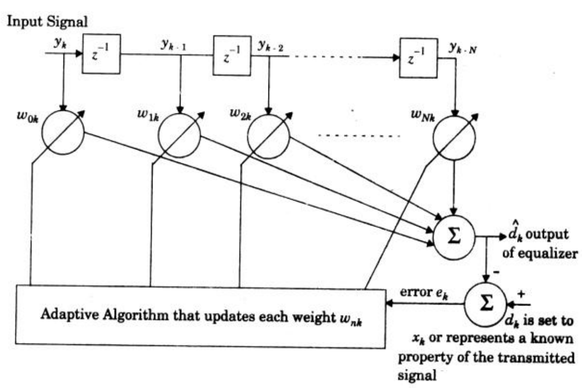

Fig. 4.1 LMS Equalizer.

Referring to the figure 3.1, the prediction error is given by

$$e_k = d_k - \hat{d}_k = x_k - \hat{d}_k \tag{4.1}$$

and

$$e_k = x_k - y_k^T w_k = x_k - w_k^T y_k \tag{4.2}$$

To compute the mean square error |ek|2 at time instant k, the mean square error function is given by

$$\xi = E\left[e_k * e_k\right] \tag{4.3}$$

The LMS algorithm seeks to minimize the mean square error given in the equation (4.3).

For a specific channel condition, the prediction error ek is dependent on the tap gain vector wN, so the MSE of an equalizer is a function of wN. Let the cost function J(wN) denote the mean squared error as a function of tap gain vector wN. In order to maximize the MSE, it is required to set the derivative of the equation (4.4) to zero.

$$\frac{\partial}{\partial w_N} J(w_N) = -2p_N + 2R_{NN} w_N = 0 \qquad (4.4)$$

By simplifying the above equation we get,

$$R_{NN}\hat{w}_N = p_N \qquad (4.5)$$

Equation (4.5) is a classic result, and is called the normal equation, since the error is minimized and is made orthogonal (normal) to the projection related to the desired signal *Xk*. When equation (4.5) is satisfied, the MMSE of the equalizer is

$$J_{opt} = \mathrm{J}(\hat{\mathrm{w}}_{\mathrm{N}}) = \mathrm{E}\left[\mathrm{x_k x_k}^*\right] - P^T{}_N \hat{w}_N \qquad (4.6)$$

To obtain the optimal tap gain vector ŵN, the normal equation in (4.5) must be solved iteratively as the equalizer converges to the acceptably small value of Jopt. There are several ways to do this and many variants of the LMS algorithm have been built upon the solution of equation (3.6). One obvious technique is to calculate

$$\hat{w} = R^{-1}{}_{NN} P_N \qquad (4.7)$$

However, inverting a matrix requires O(N3) arithmetic operations. Other methods such as Gaussian elimination and Cholesky factorization require O(N2) operations per iteration. The advantage of these methods which directly solve equation (4.7) is that only N symbol inputs are required to solve the normal equation. Consequently, a long training sequence is not necessary.

In practice, the minimization of the MSE is carried out recursively, and may be performed by the use of the stochastic gradient algorithm. This is more commonly called the least mean square (LMS) algorithm. The LMS algorithm is the simplest equalization algorithm and requires only 2N+1 operation per iteration. The filter weights are updated by the updated equations given below. Letting the variable n denote the sequence of iterations, LMS is computed iteratively by

$$\hat{d}_k(n) = w^T{}_N(n) y_N(n) \qquad (4.8a)$$

$$e_k(n) = x_k(n) - \hat{d}k(n) \qquad (4.8b)$$

$$w_N(n+1) = w_N(n) - ae^*{}_k(n)\, y_N(n) \qquad (4.8c)$$

where the subscript N denotes the number of delay stages in the equalizer, and α is the step size which controls the convergence rate and stability of the algorithm.

The LMS equalizer maximizes the signal to distortion ratio at its output within the constraints of the equalizer filter length. If an input signal has a time dispersion characteristic that is greater than the propagation delay through the equalizer, then the equalizer will be unable to reduce distortion. The convergence rate of the LMS algorithm is due to the fact that there is only one parameter, the step size α that controls the adaptation rate. To prevent adaptation from becoming unstable, the value of α is chosen from

$$O < \alpha < 2 / \sum_{i=1}^{N} \lambda_i \qquad (4.9)$$

where λi is the ith eigen value of the covariance matrix RNN. Since

$$\sum_{i=1}^{N} \lambda_i = y^T{}_N(n) y_N(n) \qquad (4.10)$$

5. Simulation Results

Fig. 5.1, Fig. 5.2, show the signal to noise ratio (SNR) vs Bit Error Rate (BER) curve for different modulation techniques with AWGN. It is noteworthy that the BER value keeps on decreasing with a considerable increase in SNR value, in Additive White Gaussian Noise (AWGN) Channel environments.

Fig. 5.1 shows BER values for different SNR values, for BPSK modulation which delves that while SNR increases the BER is being decreased. At the transmitting point of *0db* SNR value, it has $10^{-1.3}$ BER value and at the receiving point of *10db* SNR value, it has $10^{-5.8}$ BER value. The similar value of SNR is considered for all modulation techniques.

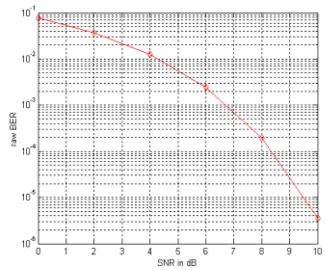

Fig.5.1.BPSK with AWGN.

Fig. 4.2 shows BER values for different SNR values, for QPSK modulation which delves that while SNR increases the BER is being decreased. At the transmitting point of *0db* SNR value, it has $10^{-0.5}$ BER value and at the receiving point of *14dB* SNR value, it has $10^{-6.3}$ BER value.

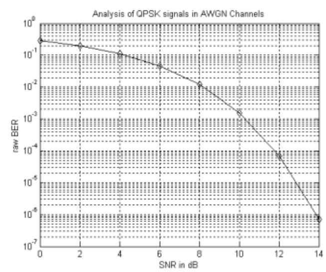

Fig. 5.2. QPSK with AWGN.

From the above two figures, we can infer that QPSK has lesser Bit Error Rate than BPSK.

Comparison of BPSK and QPSK modulation techniques

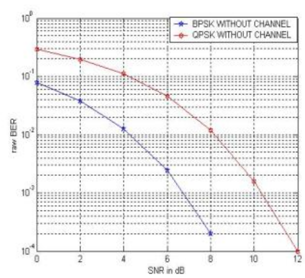

Fig.5.3 Comparative graph for QPSK and BPSK with AWGN.

Table 5.1 gives a comparison of BPSK and QPSK techniques. QPSK modulation have more BER compared to BPSK modulation.

Table 5.1. Comparison of BPSK and QPSK techniques.

S. No	Modulation Technique	SNR (at transmitter)	BER (at Transmitter)	SNR (at Receiver)	BER (at Receiver)
1	BPSK	0dB	$10^{-1.3}$	10dB	$10^{-5.8}$
2	QPSK	0dB	$10^{-0.5}$	14dB	$10^{-6.5}$

Comparison of different α value using LMS algorithms

Here, we compare the predictions of the channel coefficients using LMS algorithm. Before going to the analysis, we first applied LMS algorithm to predict Channel coefficient by using different α values. It is found that the prediction is better when we take α to be 0.0005 and hence we take α = 0.0005 for comparing predicted and original values of the channel coefficients for each of LMS algorithms. Hence we can infer that α = 0.0005 is more effective than the other α = 0.05 and α = 0.005.

Fig. 5.4 shows the original and predicted values of channel coefficients using LMS algorithm with α = 0.05. Separate comparisons are made for the real and imaginary parts of the channel coefficients. It is observed that the difference between the actual and predicted values obtained using LMS algorithm is significant and hence we can infer that the tracking may not be efficient if we use LMS algorithm.

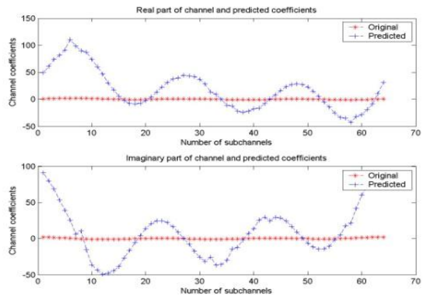

Fig. 5.4 Original and predicted channel coefficients for LMS with α= 0.05.

Fig. 5.5 gives the plot for predicted SNR and BER values, using LMS algorithm, for BPSK modulation on wireless channel with OFDM. Here, the value of α is taken as 0.05. It is observed that the achievable BER is varying dramatically with respect to perfect α value.

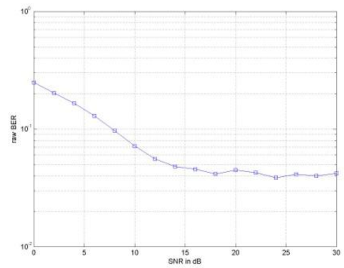

Fig. 5.5 SNR vs BER for LMS with α= 0.05.

Fig. 5.6 shows the original and predicted values of channel coefficients using LMS algorithm with α = 0.005. Separate comparisons are made for the real and imaginary parts of the channel coefficients. It is observed that the difference between the actual and predicted values obtained using LMS algorithm is significant and hence we can infer that the tracking may not be efficient if we use LMS algorithm.

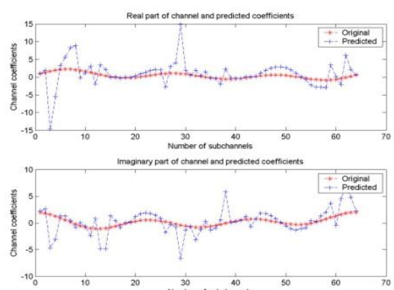

Fig. 5.7 Original and predicted channel coefficients for LMS with α= 0.005.

Fig. 5.8 gives the plot for predicted SNR and BER values, using LMS algorithm, for BPSK modulation on wireless channel with OFDM. Here, the value of α is taken as 0.005. It is observed that the achievable BER is varying dramatically with respect to perfect α value.

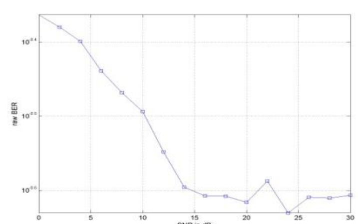

Fig. 5.8 SNR vs BER for LMS with α= 0.005.

Fig. 5.9 shows the original and predicted values of channel coefficients using LMS algorithm with α = 0.005. Separate comparisons are made for the real and imaginary parts of the channel coefficients. It is observed that the difference between the actual and predicted values obtained using LMS algorithm is significant and hence we can infer that the tracking may not be efficient if we use LMS algorithm.

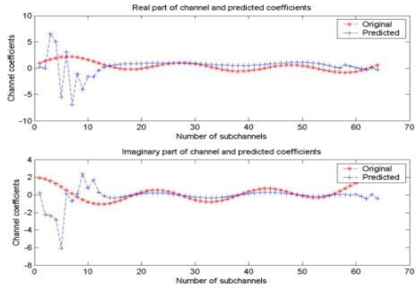

Fig. 5.9 Original and predicted channel coefficients for LMS with α= 0.0005.

Fig. 5.10 gives the plot for predicted SNR and BER values, using LMS algorithm, for BPSK modulation on wireless channel with OFDM. Here, the value of α is taken as 0.0005. It is observed that the achievable BER is varying dramatically with respect to perfect α value.

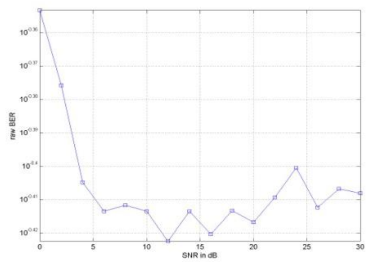

Fig. 5.10 SNR vs BER for LMS with α= 0.0005.

5. Conclusion

In this paper the Performance Evaluation of α (0.05, 0.005 and 0.0005) values for OFDM System using LMS Algorithm. These different α value is considered in this work and their performances are statistically compared by using computer simulations. Here, we compare the predictions of the channel coefficients using LMS algorithm. Before going to the analysis, we first applied LMS algorithm to predict Channel coefficient by using different α values. It is found that the prediction is better when we take α to be 0.0005 and hence we take α = 0.0005 for comparing predicted and original values of the channel coefficients for each of LMS algorithms. Hence we can infer that α = 0.0005 is more effective than the other α = 0.05 and α = 0.005.

Reference

1. **Ahn, J., Lee, H. S.(1993),** "Frequency Domain Equalisation of OFDM signals over Frequency Nonselective Rayleigh Fading Channel", *Electronics Letters*, Vol. 29, No.16, pp. 1476 – 147.**Bernard Sklar**, Rayleigh Fading Channels in Mobile Digital Communication Systems (Part I and II), July 1997.
2. **P. Schramm and R. Muller(1998),** "Pilot symbol assisted BPSK on Rayleigh fading channels with diversity: Performance analysis and parameter optimization," IEEE Transaction on communication, vol. 46, no. 12, pp. 1560–1563.
3. **Cimini, L.J.(1985),** "Analysis and simulation of digital mobile channel using orthogonal frequency division multiplexing", *IEEE Transaction on communication*, Vol.33.no.7, pp 665-675..
4. **Rappaport,T.S.,(2009)** , "Wireless Communication", Prentice Hall.
5. **S.Galih, R.Karlina, F.Nugroho, A.Irawan, T.Adiono and A. Kurniawan(2009),** "High mobility data pilot Based channel estimation for downlink OFDMA system based on IEEE 802.16e standard ",2009 international conference on Electrical Engineering and Information 5-7 Selangor, Malaysia.