A Two Bus Equivalent Method for Determination of Steady State Voltage Stability Limit of a Power System

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Abstract-
The problem of voltage instability is gaining more and more importance because of the unusual growth of power systems and insufficient or inefficient reactive power management. The voltage stability problem of a power system is associated with a rapid voltage drop due to heavy system load. Voltage reduction has a cumulative effect unless ample reactive power sources are available to regulate the voltage and maintain the reactive power balance. In this paper, simple and direct method of determining the steady state voltage stability limit of a power system at a particular load bus is implemented. The maximum permissible loading of a particular load bus is determined through a simplified Two-Bus equivalent model, called "Thevenin's Equivalent" of the original system. This method uses the base-case system information to find special Two-Bus equivalents of the system for analyzing the voltage stability problem. The effectiveness of this method is tested on a simple Two-Bus system and on the IEEE 14 Bus and IEEE 30 Bus systems and the results are compared with Newton- Raphson method. System performance is analyzed with and without a Static Var Compensator (SVC). The effects of load power factor and SVC rating on voltage stability limit are also studied. This method is very simple and does not require repetitive load flow simulations

Index Terms-Voltage Stability, Y-bus, Newton-Raphson Load Flow, Thevenin's Equivalent circuit, Q-V curves, Static Var Compensator (SVC)

I. Introduction

Power utilities are now forced to increase the utilization of existing transmission facilities to meet the growing demand without constructing new lines that are not only expensive but also environmentally unfriendly. Therefore transmission lines in a power systems are loaded more heavily than ever before to avoid the capital cost of building new lines. A voltage collapse can take place in systems or subsystems and can appear quite abruptly.

When a system approaches the voltage collapse point, the voltage magnitude of some critical buses decreases rapidly with the increase of load. Controls or operators may not be able to prevent the voltage decay, sometimes may aggravate the situation, which results in voltage collapse. Voltage collapse has become an increasing threat to power system security and reliability. Many incidents of system blackouts because of voltage stability problems have been reported worldwide. Determination of steady state voltage stability limit is thus very important in order to operate the system with an adequate stability margin.

Nowadays, a proper analysis of the voltage stability problem has become one of the major concerns in power system operation and planning studies. The main reason for voltage instability in a power system is inadequate reactive power support at some critical buses. Voltage instability is a reactive power problem. The loading of a transmission network can be increased by maintaining proper voltage profile through injecting appropriate reactive power into the system. Unlike active power, it is very difficult to estimate the reactive power margin required to achieve a certain degree of voltage security.

When the voltage of a system starts to decrease, the current, and hence the reactive power loss in transmission lines and transformers, is increased. On the other hand, a decrease in voltage reduces the reactive power supply by the line charging and shunts capacitors. Thus the voltage reduction has a cumulative effect unless ample reactive power sources or some appropriate controls are available to regulate the voltage and maintain the reactive power balance.

Ii. Methodology

The phenomenon of voltage collapse on a transmission system, due to operation near the maximum transmissible power, is characterized by a fall in voltage, which is at first gradual and then rapid. The theoretical relationship between power transferred across a system and the receiving-end voltage follows an approximately parabolic shape. These curves are usually generated from the results of repetitive load flow simulations under modified initial conditions. Once the curves are generated, the voltage stability limit can easily be determined from the "nose" point of the curves.

The process of generating the curves is very time consuming, especially for a large system. However, the computational time can significantly be reduced if the nose point can directly be determined without practically generating the curves. Direct determination of the nose point is possible if the power system can faithfully be represented by an equivalent Two-Bus system. The maximum loading capability of a particular load bus in a power system is determined through the Thevenin's equivalent circuit.

The Thevenin's equivalent circuits of all load buses are obtained in a single shot. Special care has been taken in modeling the generators to reflect actual operation, even for a change in operating conditions. This approach can provide very good results with less computation using the base-case system information. Note that the operating point of the generators at the verge of voltage stability may differ significantly from the base-case operating point. Thus, the Thevenin's equivalent circuit obtained at the base case with a conventional generator model may not represent a good equivalent circuit to determine the voltage stability limit unless some special care is taken in modeling the generators.

Iii. Fast Method For Finding Thevenin's Equivalent Circuit

A very fast approach to determine the Thevenin's equivalent circuits of all load buses in a single shot is implemented in the following sections. This approach uses the results of a single load flow solution and the system Z matrix. Both the load flow solution and the Z matrix are obtained by considering all the loads in the system. The voltage and impedance of the Thevenin's equivalent circuit are then obtained by slightly modifying the load flow solution and the diagonal elements of the Z matrix in order to nullify the effects of load impedance at the candidate bus.

Determination of the Thevenin's Impedance:

Let Z_{kk} be the k_{th} diagonal element of the Z matrix when all loads are considered. Our aim is to find the Thevenin's impedance Z_{th} of bus k when its load is ignored. These two impedances (Z_{kk} and Z_{th}) are shown in Fig 1.

It can be observed in the Fig 1 that

$$Z_{kk} = Z_k^L \text{ parallel with } Z_{th} = \frac{Z_k^L Z_{th}}{(Z_k^L + Z_{th})}$$

Here Z_k^L is the load impedance of bus k Thus, the Thevenin's impedance Z_{th} can readily be written as

$$Z_{th} = \frac{1}{\left(\frac{1}{Z_{kk}} - \frac{1}{Z_k^L}\right)} \qquad [1]$$

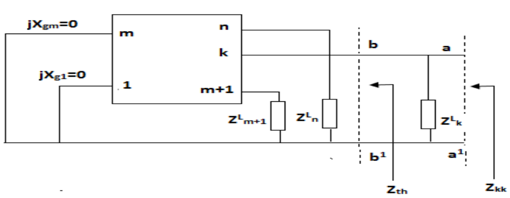
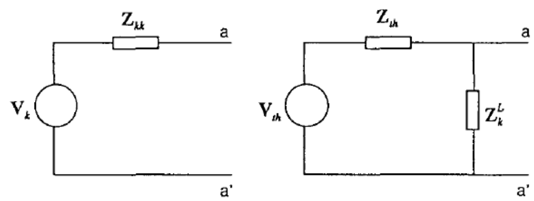


Fig 1. Thevenin's Impedance of Load Bus k.

Determination of the Thevenin's Voltage:

Let V_k be the voltage at bus k obtained from the load flow solution when all loads in the system are considered. The objective is to find the Thevenin's voltage V_{th} of bus k when its load is ignored. Fig 2 (a) and (b) shows the Thevenin's equivalent circuits of Fig 1at points aa' and bb', respectively.



(a) (b)

Fig. 2. Thevenin's Equivalent Circuits of Load Bus k

By comparing Figures 2 (a) and (b), the value of V_{th} can readily be written as

$$V_{th} = \left(1 + \frac{Z_{th}}{Z_k^L}\right) V_k \qquad [2]$$

Fig 2(b) represents the Thevenin's equivalent circuit of bus k and the maximum loading capability of this bus can be determined by varying the load impedance Z_k^L.

Iv. Voltage Stability Limit Of A Simple Two-Bus System

Consider a simple Two-Bus system as shown in Fig 3. The generator at bus 1 transfers power through a transmission line having an impedance of Z = R +jX to a load center at bus 2. Bus 1 is considered as a swing bus where both the voltage magnitude V2 and angle δ1, are kept constant.

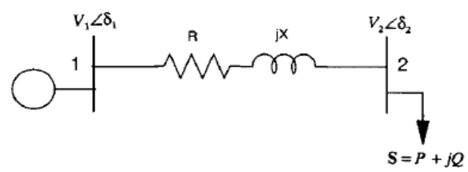


Fig 3. A simple Two-Bus system

For a given value of V_1 the relationship between the load voltage magnitude V_2 and the load power S = P +jQ can readily be written as

$$V_1 = V_2 + IZ = V_2 + I\sqrt{R^2 + X^2}$$

$$V_1^2 = V_2^2 + 2(RP + XQ) + \frac{P^2 + Q^2}{V_2^2}(R^2 + X^2) \qquad [3]$$

$$V_2^4 + [2(RP + XQ) - V_1^2]V_2^2 + [(R^2 + X^2)(P^2 + Q^2)] = 0$$

By assuming $x = V_2^2$, the above equation can be written in quadratic form as follows

$$a_1 x^2 + b_1 x + c_1 = 0 \qquad [4]$$

Where

$a_1 = 1;\ b_1 = 2(RP + XQ) - V^2;$

$c_1 = (R^2 + X^2)(P^2 + Q^2)$

The positive voltage magnitudes of bus 2 can be obtained from the solution of equation [4] and are given by

$$V_2^2 = \frac{-b_1 \pm \sqrt{{b_1}^2 - 4a_1c_1}}{2a_1}$$

$$V_2^H = \sqrt{\frac{-b_1 + \sqrt{d}}{2a_1}} \qquad [5a]$$

$$V_2^L = \sqrt{\frac{-b_1 - \sqrt{d}}{2a_1}} \qquad [5b]$$

Where the discriminant'd' is given by

$$d = b_1^2 - 4a_1c_1$$

$$d = V_1^4 + 4[2PQRX - (RP + XQ)V_1^2 - (R^2Q^2 + X^2P^2)] \qquad [6]$$

Here, V_2^H is called the high-voltage or stable solution while V_2^L is called the low-voltage or unstable solution. For zero load (P = Q = 0), V_2^H and V_2^L become V_1 and 0 respectively. As the load (at normal power factor) is increased from zero, V_2^H decreases while V_2^L increases. This process continues until a point is reached where both V_2^H and V_2^L become the same. This occurs when the value of d in equation [6] becomes zero. The load power for which $V_2^H = V_2^L$ is called the critical power and the corresponding voltage is called the critical voltage. It is said that the system has reached the voltage stability limit and it is not capable of transferring any additional power. For higher load power, the real solution of equation [4] (and hence the magnitude of V_2) will cease to occur because of the negative value of d.

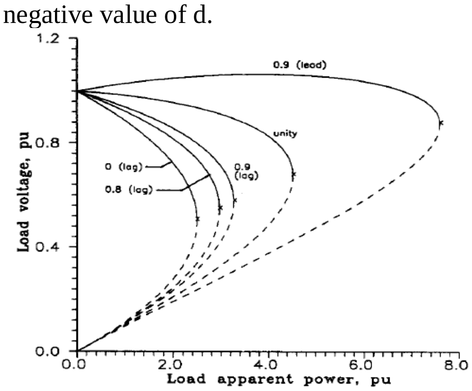


Fig 4. Variation of Load Voltage against the Load Apparent Power for various Power Factors

Typical variations of load voltage against the load apparent power for various power factors are shown in Figure 3.2 which is plotted for V1 = 1.0 p.u., R = 0.01 p.u. and X = 0.1 p.u. The high-voltage or stable solution is represented by full curves while the low-voltage or unstable solution is represented by broken curves. These two curves or voltages meet at the critical point. It can be observed in the figure that both the maximum load apparent power and critical voltage increase as the load PF changes from lagging to leading.

Critical Load Apparent Power:

Obviously, the condition of the maximum load apparent power (S_m) can be obtained by setting the value of d in Eq. (4) to zero. This gives the following quadratic equation:

$$a_2S_m^2 + b_2S_m + c_2 = 0 \qquad [7]$$

Where

$a_2 = 4[RXsin2\theta - R^2sin^2\theta - X^2cos^2\theta]$

$b_2 = -4V_1^2(Rcos\theta + Xsin\theta)$ and $c_2 = V_1^4$

In deriving the above equation, it is considered that P = S cosθ and Q = S sin θ, where θ is the PF angle. The value of S_m can be obtained from the solution of above equation

$$S_m = \frac{V_1^2}{2}\,\frac{Z - (Rcos\theta + Xsin\theta)}{(Rsin\theta + Xcos\theta)^2} \qquad [8]$$

Here $Z = \sqrt{(R^2 + X^2)}$

V. Voltage Stability Limit Of A Simple Two-Bus System With S.V.C

This Section describes the technique of directly determining the voltage stability limit or noise point of the P- V curve of a simple Two-Bus system of Fig 5. The system transfers power from a generating station to a load center through a transmission line. A SVC of finite reactive power rating is also placed at the load center. The SVC is usually connected through a step down transformer as shown in Fig 5. It consists of a fixed capacitor C and a thyristor controlled inductor L. The reactive power of the SVC can be adjusted by controlling the firing angle of the thyristors.

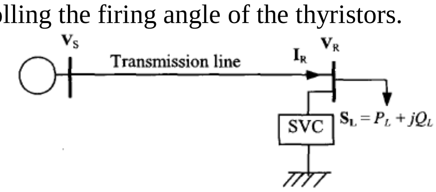


Fig 5. Single Line Diagram of a Simple Two-Bus System with SVC

When the load increases, the receiving end voltage of the line decreases and the SVC injects capacitive reactive power to boost the voltage. However, when the operation of the SVC hits the upper limit, it cannot adjust the reactive power anymore to maintain the desired voltage. Thus the load voltage decreases with further increase in load and the ultimate result is voltage collapse. It may be mentioned here that the voltage collapse may not occur until the operation of the SVC reaches the upper limit. For such an operation, the SVC can be represented by a fixed capacitive susceptance B_c

In this case, the current (I_R) at the receiving end of the line can be written as

$$I_R = \left(\frac{S_L}{V_R}\right)^* + jB_C V_R \qquad [9]$$

Here SL (= $P_L + jQ_L$) is the complex load and VR is the complex receiving end voltage. The sending end voltage Vs of the system can be written as

$$V_S = AV_R + BI_R \qquad [10]$$

Here A and B are the transmission line constants. In rectangular form, A and B can be expressed as

$$A = a_1 + ja_2 \text{ and } B = b_1 + jb_2 \qquad [11]$$

Using above the sending end voltage of equation can be written as

$$V_S = (a_1 + ja_2)V_R + (b_1 + jb_2)\left(\left(\frac{S_L}{V_R}\right)^* + jB_C V_R\right)$$

$$V_S = \left[c_1 V_R + (b_1\cos\theta + b_2\sin\theta)\frac{S}{V_R}\right] + j\left[c_2 V_R + \frac{S}{V_R}(b_2\cos\theta - b_1\sin\theta)\right] \qquad [12]$$

Where $c_1 = a_1 - b_2 B_c$ and $c_2 = a_1 + b_1 B_c$

In deriving equation [12], V_R is considered as a reference. Then

$$c_3 V_R^4 + c_4 V_R^2 + c_5 = 0 \qquad [13]$$

Where $c_3 = (c_1^2 + c_2^2)$

$$c_4 = \{2S[(b_1 c_1 + b_2 c_2)\cos\theta + (b_2 c_1 - b_1 c_2)\sin\theta - V_S^2]\}$$

$$c_5 = S^2(b_1^2 + b_2^2)$$

Note that equation [13] has four possible solutions but only the feasible solutions (real and positive) can be used to generate the P-V curve of the system.

The main objective is to directly determine the load apparent power at the nose point of the P-V curve without practically generating the curve. At the nose point or the same solution (stable and unstable) of load voltage magnitude, the coefficients of equation [11] must satisfy the following criterion

For critical value of Sm,

$$c_4^2 - 4c_3 c_5 = 0 \qquad [14]$$

The above non-linear equation can be expressed as

$$\int(S,\theta) = 0 \qquad [15]$$

$$S_m = \frac{c_7 \pm \sqrt{c_7^2 - 4c_6 c_8}}{2c_6}$$

Where

$$c_6 = 4[(b_1 c_1 + b_2 c_2)\cos\theta + (b_2 c_1 - b_1 c_2)\sin\theta]^2 - (c_1^2 + c_2^2)(b_1^2 + b_2^2)$$

$$c_7 = -4V_S^2[(b_1 c_1 + b_2 c_2)\cos\theta + (b_2 c_1 - b_1 c_2)\sin\theta]$$

$$c_8 = V_S^4$$

For a given load power factor angle θ, equation [15] can be expressed by a second order polynomial of load apparent power S and the feasible solution of the polynomial can be considered as the critical load apparent power Scr at the nose point of the P-V curve.

Vi. Test Cases And Simulation Results

In the NR method, the maximum apparent power loading of a load bus is determined by gradually increasing the power demand at the candidate bus in the original unreduced system until the method fails to converge in solving the load flow problem. As mentioned earlier, the maximum loading capability of a bus estimated by the Two-Bus method is slightly higher than the corresponding actual value because of the constant-impedance load model. Thus, the results obtained by the Two-Bus and NR methods and the actual value can be ranked as follows:

Results obtained by
NR method < Actual value < Two-Bus method

The IEEE-14 Bus System:

Using Newton-Raphson and Thevenin's (or Two–Bus) Equivalent circuit methods, Load Voltage versus Load Apparent Power curves are drawn for IEEE-14 Bus by varying Load apparent power at any bus and the Maximum Load Apparent Power values with both the methods are compared [Table 1]. Variation of Load Voltage against Load Apparent Power of 12 and 14 Buses of IEEE-14 Bus system with NR and Two-Bus methods is shown in Fig 6 and Fig 7. In this case, the system is first represented by an equivalent Two-Bus system.

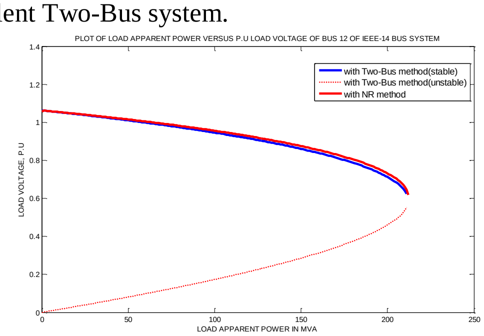


Fig 6. At Bus 12 of IEEE-14 Bus system

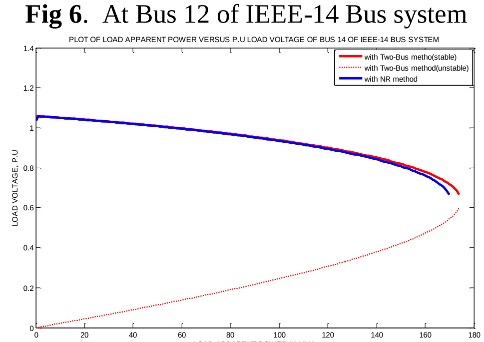


Fig 7. At Bus 14 of IEEE-14 Bus system

The critical load apparent power of the bus is then determined and the variation of critical load apparent power against the load power factor angle is shown in Fig 8. The voltage stability limit of bus 14 of the system is also determined from repetitive load flow simulations using Newton-Raphson method for comparison.

It can be noticed from Table 1 that the results (apparent power at the voltage collapse point) obtained by the load flow simulations (Newton-Raphson method) are slightly lower than the corresponding values found by the Two-Bus method and maximum error that observed at the 14 bus IEEE-14 Bus system is 3.6% at a load power factor of unity and when the system is equipped with SVC of 1.6 p.u.

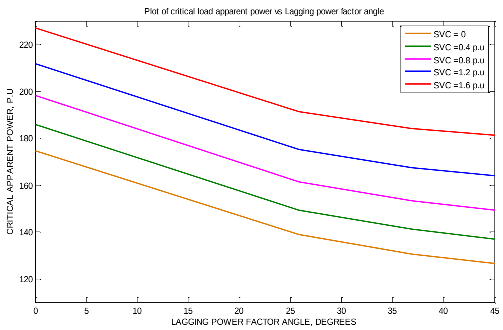


Fig 8. Variation of Critical Load Apparent Power of Bus 14 of IEEE-14 Bus System

Table 1

Comparison of Results obtained by Two-Bus and NR Methods of Bus14 of IEEE-14 Bus System

Svc Value In P.U	Power Factor (Lagging)	Critical Apparent Power (MVA)		%Error
		Two-Bus Method	NR Method	
0	1	174.64	170	2.73
	0.9	138.84	136	2.09
	0.8	130.61	129	1.25
	0.7	126.21	124	1.79
0.4	1	185.77	181	2.64
	0.9	149.36	147	1.61
	0.8	141.09	139	1.50
	0.7	136.78	135	1.32
0.8	1	198.13	193	2.66
	0.9	161.42	159	1.52
	0.8	153.24	151	1.48
	0.7	149.12	147	1.44
1.2	1	211.82	205	3.32
	0.9	175.30	172	1.92
	0.8	167.41	165	1.46
	0.7	163.65	162	1.02
1.6	1	226.88	219	3.60
	0.9	191.32	188	1.77
	0.8	184.04	181	1.68
	0.7	180.90	179	1.06

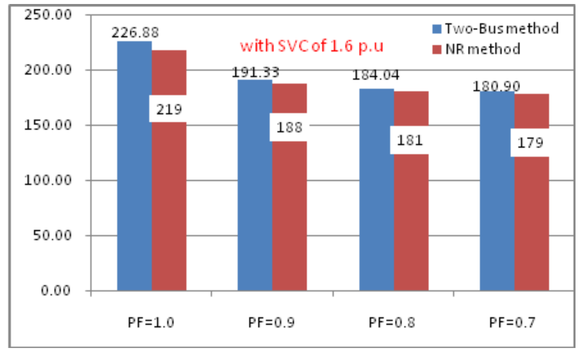


Fig 9. Variation of Critical Load Apparent Power of Bus 14 of IEEE-14 Bus System for different Power Factors with SVC of 1.6 p.u

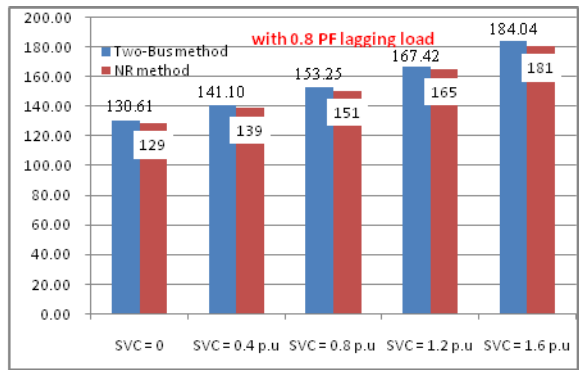


Fig 10. Variation of Critical Load Apparent Power of Bus 14 of IEEE-14 Bus System for different values of SVC with 0.8 PF Lagging Load

The IEEE-30 Bus System:

The load voltage versus load apparent power curves of 20and 24 buses of IEEE-30 Bus system are shown in Figures 11 and 12 respectively. From all these figures it is clearly shown that the critical apparent power at any load bus obtained by NR method and Two-bus methods are very close.

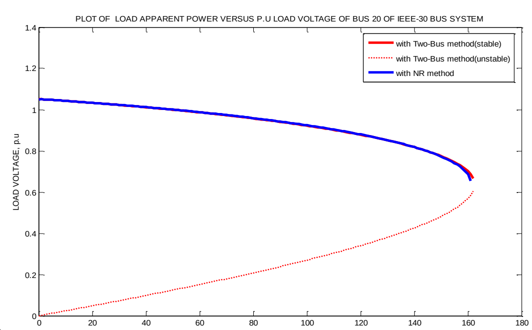


Fig 11. AtBus 20 of IEEE -30 Bus System

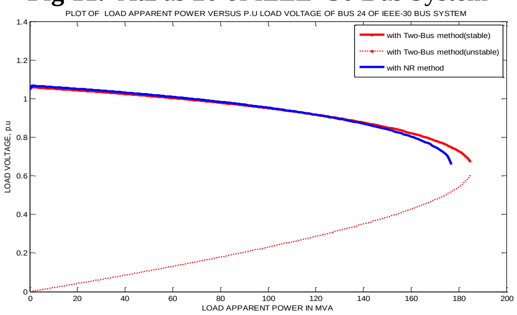


Fig 12. At Bus 24 of IEEE -30 Bus System

For the IEEE-30 Bus system, the voltage stability limit of bus 24 is determined. Table 2 summarizes the results obtained by the Two-Bus method as well as the repetitive load flow simulations. Maximum error that observed at the 24 bus IEEE-30 Bus system is 5.28% at a load power factor of unity and when the system is equipped with SVC of 1.6 p.u.

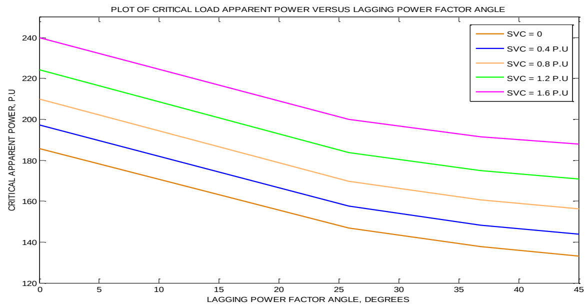


Fig 13. Variation of Critical Load Apparent Power against Load Power Factor Angle of Bus 24 of IEEE-30 Bus System for various values of SVC

Table 2

Comparison of Results obtained by Two-Bus and Newton-Raphson Methods of Bus 24 of IEEE-30 Bus System

Svc Value In P.U	Power Factor (Lagging)	Critical Apparent Power (MVA)		%Error
		Two-Bus Method	NR Method	
0	1	185.69	177	4.90
	0.9	146.72	143	2.60
	0.8	137.70	135	2.00
	0.7	132.84	131	1.40
0.4	1	197.23	188	4.90
	0.9	157.45	153	2.90
	0.8	148.34	145	2.30
	0.7	143.52	142	1.07
0.8	1	210.06	200	5.03
	0.9	169.72	165	2.86
	0.8	160.62	157	2.30
	0.7	155.92	154	1.24
1.2	1	224.30	213	5.30
	0.9	183.81	179	2.68
	0.8	174.88	172	1.67
	0.7	170.46	168	1.46
1.6	1	240.05	228	5.28
	0.9	200.06	195	2.59
	0.8	191.57	188	1.89
	0.7	187.65	185	1.43

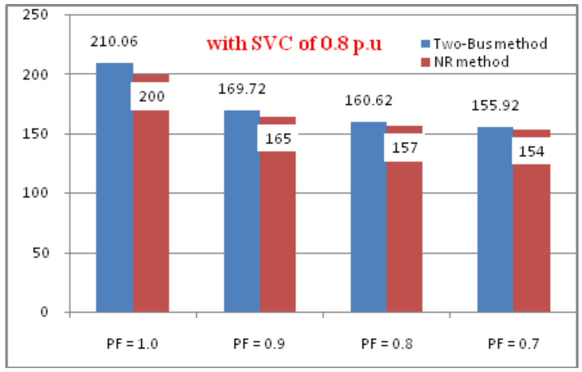


Fig 14. Variation of Critical Load Apparent Power of Bus 24 of IEEE-30 Bus System for different Power Factors with SVC of 0.8 p.u

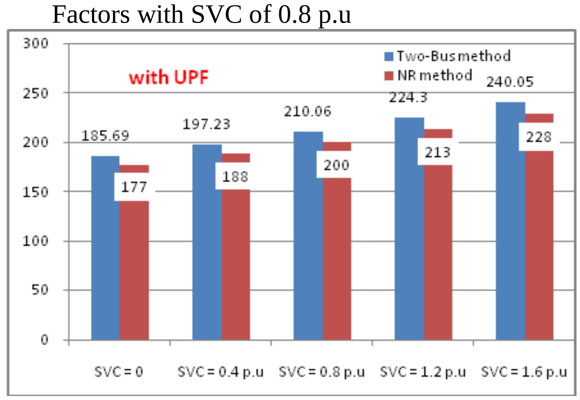


Fig 15. Variation of Critical Load Apparent Power of Bus24 of IEEE-30 Bus System for different values of SVC with UPF Load

Conclusion

A simple and fast method for analyzing the voltage stability problem of a general power system through a two-bus equivalent has been implemented. The generator model used in this project is very insensitive to the change in operating conditions. Thus the two-bus equivalents obtained at the base-case operating point through the Thevenin's theorem can be faithfully applied to determine the steady state voltage stability limit. Unlike the other methods, the Thevenin's equivalent circuits of all load buses are efficiently obtained in a single shot. This requires the results of the base-case load flow solution and computation of the Z matrix when all loads in the system are considered. A minor modification to the bus voltages and the diagonal elements of the Z matrix is required to exclude the effects of the load at the candidate bus.

Determination of the various quantities at the verge of voltage stability involves the solution of simple quadratic equations. The maximum demand at a load bus to ensure a minimum specified voltage can also be determined from the solution of another quadratic equation. This two-bus method has been tested on the IEEE 14-, and 30-Bus systems for a number of cases with and without SVC. The effects of the SVC rating and load power factor on the voltage stability limit are also studied in detail. The results obtained by this two-bus method are compared with those found by the conventional repetitive load flow simulations. Because of the constant-impedance model of the system load, the two-bus method provides slightly overestimated results at the verge of voltage stability. The convergence problem of the load flow method in the vicinity of the voltage collapse point is believed to be the main reason for the above discrepancy. The errors found in the simulation results are higher at the verge of voltage stability. Unlike the other methods, the two-bus method can provide much better and reliable results using the base-case system information with significantly less computation.

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