Fuzzy Programming approach for Fractional Multi-objective Transportation Problem with Impurity Constraints

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Abstract
In this paper, a Fractional Multi-objective Transportation Problem with Impurity Constraints having demand and supply constraints somewhat uncertain imprecise and vague in nature is formulated as Fractional Multi-objective Transportation Problem with Impurity Constraints with extreme tolerances. By using suitable transformation, an equivalent Multi-objective Linear Transportation Problem with Impurity Constraints is formulated which is transformed into an equivalent crisp model to determine an optimal solution of Fractional Multi-objective Transportation Problem with Impurity Constraints.

Keywords: Fractional Transportation Problem, Fuzzy Programming, Crisp Model, Membership Function, Multi-objective Decision Making.

1. Introduction

Conventional optimization methods assume that all parameters and goals of an optimization model are precisely known. But for many practical problems there are incompleteness and unreliability of input information. This is caused to use fuzzy multi-objective optimization method with fuzzy parameters. Bit et al. [2] presented an application of fuzzy linear programming to the linear multi-objective transportation problem, a special type of vector minimum problem in which constraints were all equality type and the objectives were conflicting in nature. Li and Lai [4] presented a fuzzy compromise programming approach to multiobjective transportation problems. A characteristic feature of the approach proposed was that various objectives were synthetically considered with the marginal evaluation for individual objectives and the global evaluation for all objectives. Verma et al. [7] used a special type of non-linear (hyperbolic and exponential) membership functions to solve the multi-objective transportation problem. Sakawa et al. [5] discussed a two objective transportation problem, minimizing the transportation cost and minimizing the opportunity loss with respect to transportation time, in a housing material manufacturer and derived a satisfactory solution to the problem using interactive fuzzy programming method.

This paper presents a Fuzzy Programming approach to solve Fractional Multi-objective Transportation Problem with Impurity Constraints. The fractional multi-objective transportation problem with impurity constraints having demand and supply constraints somewhat uncertain imprecise and vague in nature is formulated as fractional multi-objective transportation problem with impurity constraints with extreme tolerances. By using suitable transformation, an equivalent multi-objective linear transportation problem with impurity constraints is formulated which is transformed into an equivalent crisp model to determine an optimal solution of fractional multi-objective transportation problem with impurity constraints.

2. Mathematical Formulation

The mathematical formulation of Fractional Multi-objective Transportation Problem with Impurity Constraints is written as:

P_1 Maximize $$Z^G(x) = \frac{N^G(x)}{D^G(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^G x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^G x_{ij}}$$

subject to $\sum_{j=1}^{n} x_{ij} = \tilde{a}_i \qquad (i = 1,2,\ldots\ldots,m)$

$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j \qquad (j = 1,2,\ldots\ldots,n)$$

$$\sum_{i=1}^{m} f_{ijk} x_{ij} \le \tilde{q}_{jk} \qquad (j = 1,2,\ldots\ldots,n; k = 1,2,\ldots\ldots,P)$$

$$x_{ij} \ge 0 \qquad (i = 1,2,\ldots\ldots,m; j = 1,2,\ldots\ldots,n)$$

where

$Z^G(x) = [Z^1(x), Z^2(x),\ldots\ldots, Z^g(x)]$, is a vector of g fractional objective functions and the superscript on both fractional objective functions $Z^G(x)$, numerator $N^G(x)$ and denominator $D^G(x)$ are used to identify the number of fractional objective functions $(G = 1,2,\ldots\ldots,g)$

$\tilde{a}_i$ = amount available is somewhat uncertain/non-stochastic imprecise and vague in nature at the i^{th} supply point

$\tilde{b}_j$ = requirement is somewhat uncertain/non-stochastic imprecise and vague in nature at the j^{th} demand point

x_{ij} = amount of commodity to be transported from the i^{th} supply point to the j^{th} demand point

f_{ijk} = units of P impurities $(k = 1,2,\ldots\ldots,P)$ in one unit of the commodity when it is transported from the i^{th} supply point to the j^{th} demand point

$\tilde{q}_{jk}$ = units of greatest quantity of impurity k that can be received by demand point j is uncertain, imprecise and vague in nature

c_{ij}^G / d_{ij}^G = the proportional contribution to the value of G^{th} fractional objective function of transporting one unit of commodity from the i^{th} supply point to the j^{th} demand point.

Assumptions:

1. Without loss of generality it will be assumed that $\tilde{a}_i > 0 \;\forall\; i$; $\tilde{b}_j > 0 \;\forall\; j$.
2. For consistency, total demand requirement equals the total supply capacity.
3. Positive Triangular Numbers
   - $\tilde{a}_i = (a_i - a_i^l, a_i, a_i + a_i^r)$ with tolerances $a_i^l (< a_i)$, $a_i^r (> 0)$.
   - $\tilde{b}_j = (b_j - b_j^l, b_j, b_j + b_j^r)$ with tolerances $b_j^l (< b_j)$, $b_j^r (> 0)$.
   - $\tilde{q}_{jk} = (q_{jk} - q_{jk}^l, q_{jk}, q_{jk} + q_{jk}^r)$ with tolerances $q_{jk}^l (< q_{jk})$, $q_{jk}^r (> 0)$.
4. $\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^G x_{ij} > 0$, for all feasible solution.

In this context, it may be noted that the impurity constraints of P_1 can be written as:

$$\sum_{i=1}^{m} f_{ijk} x_{ij} + x_{M+k,j} = \tilde{q}_{jk}$$

$$x_{M+k,j} \ge 0$$

where $x_{M+k,j}$ are the slack variables to the impurity constraints.

P_1 can be modified as the Fractional Multi-objective Transportation Problem with Impurity Constraints with extreme tolerances:

P_2 Maximize $Z^G(x) = \frac{N^G(x)}{D^G(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^G x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^G x_{ij}}$

subject to

$$\sum_{j=1}^{n} x_{ij} \geq a_i - a_i^l$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i + a_i^r$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j - b_j^l$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j + b_j^r$$

$$\sum_{i=1}^{m} f_{ijk} x_{ij} + x_{M+k,j} \geq q_{jk} - q_{jk}^l$$

$$\sum_{i=1}^{m} f_{ijk} x_{ij} + x_{M+k,j} \leq q_{jk} + q_{jk}^r \qquad x_{ij} \geq 0, x_{M+k,j} \geq 0$$

With the help of transformation $y = tx$, $(t > 0)$ [1], an equivalent Multi-objective Linear Transportation Problem with Impurity Constraints may be written as:

P_3 Maximize $[t \cdot N^G(y/t)]$

subject to

$$t \cdot D^G(y/t) \leq 1$$

$$\frac{1}{t}\sum_{j=1}^{n} y_{ij} \geq a_i - a_i^l$$

$$\frac{1}{t}\sum_{j=1}^{n} y_{ij} \leq a_i + a_i^r$$

$$\frac{1}{t}\sum_{i=1}^{m} y_{ij} \geq b_j - b_j^l$$

$$\frac{1}{t}\sum_{i=1}^{m} y_{ij} \leq b_j + b_j^r$$

$$\frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] \geq q_{jk} - q_{jk}^l$$

$$\frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] \leq q_{jk} + q_{jk}^r$$

$$y \geq 0, \quad y' \geq 0, \; t > 0$$

The fuzzy objective functions and constraints are characterized by their membership functions. To optimize the objective functions and the constraints, a decision in a fuzzy environment is defined in analogy to nonfuzzy environments as the selection of activities which simultaneously satisfy objective functions and constraints. Therefore, the decision in a fuzzy environment can be viewed as the intersection of fuzzy constraints and fuzzy objective functions.

The membership function of each objective functions and constraints can be written as:

$$\mu_G(t\cdot N^G(y/t)) = \begin{cases} 0 & if \;\; t\cdot N^G(y/t)\le 0, \\ \dfrac{t\cdot N^G(y/t)-0}{\tilde{Z}^G-0} & if \;\; 0 < t\cdot N^G(y/t) < \tilde{Z}^G, \\ 1 & if \;\; t\cdot N^G(y/t)\ge \tilde{Z}^G \end{cases}$$

$$\mu(\tilde{a}_i) = \begin{cases} 0 & if \;\; a_i > \frac{1}{t}\sum_{j=1}^{n} y_{ij} + a_i^l \\ \dfrac{\frac{1}{t}\sum_{j=1}^{n} y_{ij} + a_i^l - a_i}{a_i^l} & if \;\; \frac{1}{t}\sum_{j=1}^{n} y_{ij} < a_i \le \frac{1}{t}\sum_{j=1}^{n} y_{ij} + a_i^l \\ \dfrac{a_i - \frac{1}{t}\sum_{j=1}^{n} y_{ij} + a_i^r}{a_i^r} & if \;\; \frac{1}{t}\sum_{j=1}^{n} y_{ij} - a_i^r \le a_i < \frac{1}{t}\sum_{j=1}^{n} y_{ij} \\ 0 & if \;\; a_i \le \frac{1}{t}\sum_{j=1}^{n} y_{ij} - a_i^r \end{cases}$$

$$\mu(\tilde{b}_j) = \begin{cases} 0 & if \;\; b_j > \frac{1}{t}\sum_{i=1}^{m} y_{ij} + b_j^l \\ \dfrac{\frac{1}{t}\sum_{i=1}^{m} y_{ij} + b_j^l - b_j}{b_j^l} & if \;\; \frac{1}{t}\sum_{i=1}^{m} y_{ij} < b_j \le \frac{1}{t}\sum_{i=1}^{m} y_{ij} + b_j^l \\ \dfrac{b_j - \frac{1}{t}\sum_{i=1}^{m} y_{ij} + b_j^r}{b_j^r} & if \;\; \frac{1}{t}\sum_{i=1}^{m} y_{ij} - b_j^r \le b_j < \frac{1}{t}\sum_{i=1}^{m} y_{ij} \\ 0 & if \;\; b_j \le \frac{1}{t}\sum_{i=1}^{m} y_{ij} - b_j^r \end{cases}$$

$$\mu(\tilde{q}_{jk}) = \begin{cases} 0 & if \;\; q_{jk} > \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] + q_{jk}^l \\ \dfrac{\frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] + q_{jk}^l - q_{jk}}{q_{jk}^l} & if \;\; \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] < q_{jk} \le \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] + q_{jk}^l \\ \dfrac{q_{jk} - \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] + q_{jk}^r}{q_{jk}^r} & if \;\; \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] - q_{jk}^r \le q_{jk} < \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] \\ 0 & if \;\; q_{jk} \le \frac{1}{t}\left[\sum_{i=1}^{m} f_{ijk} y_{ij} + y'\right] - q_{jk}^r \end{cases}$$

3. Crisp Model

By introducing an auxiliary variable λ, P_3 can be transformed into the following Crisp Model [3]:

P_4 Max λ

subject to $\mu_G(t\cdot N^G(y/t)) \ge \lambda$

$$\mu(\tilde{a}_i) \ge \lambda$$

$$\mu(\tilde{b}_j) \ge \lambda$$

$$\mu(\tilde{q}_{jk}) \ge \lambda$$

$$t\cdot D^G(y/t) \le 1$$

$$0 \le \lambda \le 1,\; t > 0$$

The constraints in P_4 containing cross product terms $\lambda\, t$ which are not convex. Therefore the solution of this problem requires the special approach adopted for solving general non-convex application problems and therefore is solved by fuzzy decisive set method [6].

4. Algorithm

The steps of the algorithm are as follows:

Step 1: Formulate Problem P_1 as P_2.

Step 2: Obtain an equivalent Multi-objective Linear Transportation Problem with Impurity Constraints: P_3 using transformation $y = tx$, $(t > 0)$.

Step 3: Determine maximum aspiration level $\overline{Z}^G$ by maximizing each objective function of P_3.

Step 4: Define membership function of each objective function, constraints and impurity constraints of P_3.

Step 5: Transform P_3 into an equivalent crisp model P_4 by introducing an auxiliary variable λ.

Step 6: Solve the transformed crisp model: P_4 by using fuzzy decisive set method and obtain the optimal value λ^* of λ.

Step 7: Obtain optimal solution of P_1 with the help of the maximum value λ^* of P_4.

5. Numerical Example

Consider the following Fractional Multi-objective Transportation Problem with Impurity Constraints. Here supplies and demands are trapezoidal fuzzy numbers. If x_{ij} be the tonnage sent from source i to destination j, then it is required to

$$\text{Max}\, Z^G(x) = \frac{\sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}^G x_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{3} d_{ij}^G x_{ij}}$$

subject to,

$$\sum_{j=1}^{3} x_{ij} = \tilde{a}_i$$

$$\sum_{i=1}^{3} x_{ij} = \tilde{b}_j$$

$$\sum_{i=1}^{3} p_i x_{ij} \le L_j \tilde{b}_j$$

$$x_{ij} \ge 0$$

$$(i = 1,2,3;\ j = 1,2,3; G = 1,2,3)$$

Source i	Destination j $[c_{i1}\ \ d_{i1}]$	$[c_{i2}\ d_{i2}]$	$[c_{i3}\ d_{i3}]$	a_i	p_i
1	$\begin{bmatrix}\begin{bmatrix}1\\5\\10\end{bmatrix} & \begin{bmatrix}4\\4\\12\end{bmatrix}\end{bmatrix}$	$\begin{bmatrix}\begin{bmatrix}2\\6\\3\end{bmatrix} & \begin{bmatrix}4\\5\\7\end{bmatrix}\end{bmatrix}$	$\begin{bmatrix}\begin{bmatrix}7\\2\\4\end{bmatrix} & \begin{bmatrix}3\\3\\5\end{bmatrix}\end{bmatrix}$	(7, 8, 10)	0.4
2	$\begin{bmatrix}\begin{bmatrix}1\\11\\1\end{bmatrix} & \begin{bmatrix}5\\7\\7\end{bmatrix}\end{bmatrix}$	$\begin{bmatrix}\begin{bmatrix}9\\3\\16\end{bmatrix} & \begin{bmatrix}8\\8\\14\end{bmatrix}\end{bmatrix}$	$\begin{bmatrix}\begin{bmatrix}3\\12\\1\end{bmatrix} & \begin{bmatrix}9\\6\\1\end{bmatrix}\end{bmatrix}$	(10,11,13)	0.8
3	$\begin{bmatrix}\begin{bmatrix}8\\4\\3\end{bmatrix} & \begin{bmatrix}6\\1\\6\end{bmatrix}\end{bmatrix}$	$\begin{bmatrix}\begin{bmatrix}9\\10\\4\end{bmatrix} & \begin{bmatrix}2\\3\\8\end{bmatrix}\end{bmatrix}$	$\begin{bmatrix}\begin{bmatrix}4\\2\\3\end{bmatrix} & \begin{bmatrix}5\\12\\2\end{bmatrix}\end{bmatrix}$	(8, 9, 11)	0.6
b_j	(6, 7, 9)	(4, 5, 7)	(15, 16, 18)		
L_j	0.7	0.7	0.7		

An equivalent multi-objective linear transportation problem with impurity constraints using transformation $y = tx$, $(t > 0)$ is:

$$\text{Max} \left\{ \begin{array}{l} z^1(y,t) = y_{11} + 2y_{12} + 7y_{13} + y_{21} + 9y_{22} + 3y_{23} + 8y_{31} + 9y_{32} + 4y_{33}, \\ z^2(y,t) = 5y_{11} + 6y_{12} + 2y_{13} + 11y_{21} + 3y_{22} + 12y_{23} + 4y_{31} + 10y_{32} + 2y_{33}, \\ z^3(y,t) = 10y_{11} + 3y_{12} + 4y_{13} + y_{21} + 16y_{22} + y_{23} + 3y_{31} + 4y_{32} + 3y_{33} \end{array} \right\}$$

subject to

$$4y_{11} + 4y_{12} + 3y_{13} + 5y_{21} + 8y_{22} + 9y_{23} + 6y_{31} + 2y_{32} + 5y_{33} \le 1$$
$$4y_{11} + 5y_{12} + 3y_{13} + 7y_{21} + 8y_{22} + 6y_{23} + y_{31} + 3y_{32} + 12y_{33} \le 1$$
$$12y_{11} + 7y_{12} + 5y_{13} + 7y_{21} + 14y_{22} + y_{23} + 6y_{31} + 8y_{32} + 2y_{33} \le 1$$
$$y_{11} + y_{12} + y_{13} - 7t \ \ge 0$$
$$y_{11} + y_{12} + y_{13} - 10t \le 0$$
$$y_{21} + y_{22} + y_{23} - 10t \ge 0$$
$$y_{21} + y_{22} + y_{23} - 13t \le 0$$
$$y_{31} + y_{32} + y_{33} - 8t \ \ge 0$$
$$y_{31} + y_{32} + y_{33} - 11t \le 0$$
$$y_{11} + y_{21} + y_{31} - 6t \ \ge 0$$
$$y_{11} + y_{21} + y_{31} - 9t \ \le 0$$
$$y_{12} + y_{22} + y_{32} - 4t \ \ge 0$$
$$y_{12} + y_{22} + y_{32} - 7t \ \le 0$$
$$y_{13} + y_{23} + y_{33} - 15t \ge 0$$
$$y_{13} + y_{23} + y_{33} - 18t \le 0$$
$$4y_{11} + 8y_{21} + 6y_{31} + y_{41} - 49t \ge 0$$
$$4y_{11} + 8y_{21} + 6y_{31} + y_{41} - 70t \le 0$$

$$4y_{12} + 8y_{22} + 6y_{32} + y_{42} - 28t \geq 0$$
$$4y_{12} + 8y_{22} + 6y_{32} + y_{42} - 49t \leq 0$$
$$4y_{13} + 8y_{23} + 6y_{33} + y_{43} - 105t \geq 0$$
$$4y_{13} + 8y_{23} + 6y_{33} + y_{43} - 126t \leq 0$$
$$y_{11}, y_{12}, y_{13}, y_{21}, y_{22}, y_{23}, y_{31}, y_{32}, y_{33}, y_{41}, y_{42}, y_{43} \geq 0,\ t > 0.$$

The following maximum aspiration levels are obtained by maximizing each objective function :

$$z^1(y,t) \underset{\sim}{>} 1.363057,\ z^2(y,t) \underset{\sim}{>} 2.100775 \text{ and } z^3(y,t) \underset{\sim}{>} 1.054341.$$

Using the membership functions, the above fuzzy model reduces to the following Crisp Model:

$$\text{Max } \lambda$$

subject to

$$y_{11} + 2y_{12} + 7y_{13} + y_{21} + 9y_{22} + 3y_{23} + 8y_{31} + 9y_{32} + 4y_{33} - 1.363057\,\lambda \geq 0$$
$$5y_{11} + 6y_{12} + 2y_{13} + 11y_{21} + 3y_{22} + 12y_{23} + 4y_{31} + 10y_{32} + 2y_{33} - 2.100775\,\lambda \geq 0$$
$$10y_{11} + 3y_{12} + 4y_{13} + y_{21} + 16y_{22} + y_{23} + 3y_{31} + 4y_{32} + 3y_{33} - 1.054341\,\lambda \geq 0$$
$$4y_{11} + 4y_{12} + 3y_{13} + 5y_{21} + 8y_{22} + 9y_{23} + 6y_{31} + 2y_{32} + 5y_{33} \leq 1$$
$$4y_{11} + 5y_{12} + 3y_{13} + 7y_{21} + 8y_{22} + 6y_{23} + y_{31} + 3y_{32} + 12y_{33} \leq 1$$
$$12y_{11} + 7y_{12} + 5y_{13} + 7y_{21} + 14y_{22} + y_{23} + 6y_{31} + 8y_{32} + 2y_{33} \leq 1$$
$$y_{11} + y_{12} + y_{13} \geq (7 + \lambda)t$$
$$y_{11} + y_{12} + y_{13} \leq (10 - 2\lambda)t$$
$$y_{21} + y_{22} + y_{23} \geq (10 + \lambda)t$$
$$y_{21} + y_{22} + y_{23} \leq (13 - 2\lambda)t$$
$$y_{31} + y_{32} + y_{33} \geq (8 + \lambda)t$$
$$y_{31} + y_{32} + y_{33} \leq (11 - 2\lambda)t$$
$$y_{11} + y_{21} + y_{31} \geq (6 + \lambda)t$$
$$y_{11} + y_{21} + y_{31} \leq (9 - 2\lambda)t$$
$$y_{12} + y_{22} + y_{32} \geq (4 + \lambda)t$$
$$y_{12} + y_{22} + y_{32} \leq (7 - 2\lambda)t$$
$$y_{13} + y_{23} + y_{33} \geq (15 + \lambda)t$$
$$y_{13} + y_{23} + y_{33} \leq (18 - 2\lambda)t$$
$$4y_{11} + 8y_{21} + 6y_{31} + y_{41} \geq (49 + \lambda)t$$
$$4y_{11} + 8y_{21} + 6y_{31} + y_{41} \leq (70 - 2\lambda)t$$
$$4y_{12} + 8y_{22} + 6y_{32} + y_{42} \geq (28 + \lambda)t$$
$$4y_{12} + 8y_{22} + 6y_{32} + y_{42} \leq (49 - 2\lambda)t$$
$$4y_{13} + 8y_{23} + 6y_{33} + y_{43} \geq (105 + \lambda)t$$
$$4y_{13} + 8y_{23} + 6y_{33} + y_{43} \leq (126 - 2\lambda)t$$
$$\lambda \geq 0, t > 0$$

Solve the crisp model by using the fuzzy decisive set method. The following values of λ are obtained in the next 30 iterations:

$\lambda = 0.50$; $\lambda = 0.75$; $\lambda = 0.625$; $\lambda = 0.6875$;
$\lambda = 0.65625$; $\lambda = 0.671875$; $\lambda = 0.6796875$; $\lambda = 0.67578125$;
$\lambda = 0.677734375$ $\lambda = 0.676757812$; $\lambda = 0.676269531$; $\lambda = 0.67602539$;
$\lambda = 0.67596332$; $\lambda = 0.675842285$; $\lambda = 0.675872802$; $\lambda = 0.675857543$;
$\lambda = 0.675849914$; $\lambda = 0.675846099$; $\lambda = 0.675848007$; $\lambda = 0.675847053$;
$\lambda = 0.67584753$; $\lambda = 0.675847291$; $\lambda = 0.675847172$; $\lambda = 0.675847113$;
$\lambda = 0.675847083$; $\lambda = 0.675847098$; $\lambda = 0.675847105$; $\lambda = 0.675847102$;
$\lambda = 0.675847103$; $\lambda = 0.675847104$.

Consequently, the maximum value $\lambda^* = 0.675847104$ is obtained at the 31st iteration and solution of Crisp Model is:

$y_{11}^* = 0.02474367$, $y_{13}^* = 0.03316893$, $y_{21}^* = 0.009168814$, $y_{22}^* = 0.003489797$,

$y_{23}^* = 0.05883124$, $y_{31}^* = 0.01079152$, $y_{32}^* = 0.03433357$, $y_{33}^* = 0.01297173$,

$y_{41}^* = 0.09557549$, $y_{42}^* = 0.08515277$, $y_{43}^* = 0.1535401$, $t^* = 0.00669641$,

$\lambda^* = 0.675847104$.

The optimal solution of the original problem is obtained as:

$x_{11}^* = 3.695064968$, $x_{13}^* = 04.953240617$, $x_{21}^* = 1.369213355$, $x_{22}^* = 0.521144464$,

$x_{23}^* = 8.785489538$, $x_{31}^* = 1.611538123$, $x_{32}^* = 5.127160673$, $x_{33}^* = 1.937117052$,

$x_{41}^* = 14.27264609$, $x_{42}^* = 12.71618225$, $x_{43}^* = 22.92871852$, $Z^{1^*} = 0.921218109$,

$Z^{2^*} = 1.555058607$, $Z^{3^*} = 0.712573308$.

6. Conclusion

To generate total transportation solutions for Fractional Multi-objective Transportation Problems with Impurity Constraints, an algorithm has been developed in this paper using Fuzzy Programming approach. Solving fractional multi-objective transportation models offers a more universal apparatus for a wider class of real life decision priority problems than the multi-objective transportation problems. The fractional multi-objective transportation problems results in a subset of feasible solutions from which a transportation system decision maker is sure of a most preferred solution.

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