

# Optimizing Atmospheric Scattering Model by Combining Decay Rates

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## ABSTRACT

The Atmospheric Scattering Model is a mathematical model used to describe the scattering of light as it propagates through the atmosphere by interacting with air molecules and particles. However, atmospheric scattering models in common use today tend to set the source constant to a constant for simplicity in calculating and processing data, which is not the case in vertical photography, where the source constant varies with the distance from the point source. In this paper, we found the decay of the learning rate during machine learning training and the step size of the learning rate are similar to the trend of the light source constant during the vertical photography of the atmospheric scattering model, also the decay of the light source constant with the distance of the point source in the vertical photography case is similar to the feedback reward in the Markov decision process in deep learning, then we use three attenuation models based on the atmospheric scattering model, linear decay, exponential decay and discounted future reward, to improve the accuracy of the traditional atmospheric scattering model in certain special scenarios, combine the machine learning method with the traditional physical model.

KEYWORDS: Atmospheric Scattering Model, Learning Rates, Decay Rates, Markov Decision

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#### I. INTRODUCTION

The Atmospheric Scattering Model is a mathematical model used to describe the scattering of light as it propagates through the atmosphere by interacting with air molecules and particles. It is widely used in the fields of atmospheric physics, remote sensing, agricultural, meteorology and computer graphics [1]. In the Atmospheric Scattering Model, light interacts with atmospheric components (such as air molecules, aerosol particles, clouds, etc.), resulting in processes such as light changing direction, scattering, absorption and transmission. These interactions are influenced by factors such as wavelength of light, angle of incidence, atmospheric composition and optical properties [2]. However, atmospheric scattering model in the case of vertical photography, the light source constant is often not a constant quantity, but for the convenience of calculation many times we regard it as a constant, but in a particular case it is changed with the distance from the light source, due to the similarity of its decay process. In this paper, we propose three optimization methods, linear decay, exponential decay and discounted future reward, to improve the value of the light source constant at different distances from the light source, which is shown as the Figure 1.1, and thus improve the Atmospheric Scattering Model by combining the learning rate decay method of machine learning and the reward mechanism of Markov decision in deep learning.



Figure: 1.1

# II. THE DERIVATION OF THE ATMOSPHERIC SCATTERING MODEL

The derivation of the Atmospheric Scattering Model is divided into two main parts of the calculation, the calculation of the reflected light from the target and the calculation of the scattered light from the atmosphere. The first is about the calculation of the target reflected light, reflected light in the process of propagation [3], with the increase of transmission distance light intensity gradually decay, considering the parallel beam through the atmospheric medium, assuming that the beam has a unit cross-sectional area, from the x position, each transmission distance dx intensity change amount can be expressed as:

$$\frac{dE(x,\lambda)}{E(x,\lambda)} = -\beta(\lambda)dx$$
 (Formula 2.1)

 $E(x, \lambda)$  is the light intensity after attenuation,  $-\beta(\lambda)$  is the scattering coefficient. Then integrating both sides of the equation simultaneously from x = 0 to d, when the light source is parallel, can get the reflected light from the target:

$$E(d, \lambda) = E_0(\lambda)e^{-\beta(\lambda)d}$$
 (Formula 2.2)

When the light source is a point source, we get:

$$E(d,\lambda) = \frac{l_0(\lambda)e^{-\beta(\lambda)d}}{d^2} = \frac{L_\infty\rho(x)}{d^2}e^{-\beta(\lambda)d}$$
(Formula 2.3)

The second part is the estimation of atmospheric scattered light, the atmospheric light component received by the detector mainly includes direct solar light, diffuse atmospheric light and ground reflected light. The total calculation of the atmospheric scattered light received by the detector is as follows: Volumetric micro elements:

# $dV = d\omega x^2 k\beta(\lambda) dx$ (Formula 2.4)

k is the light source constant. The light intensity after reaching the detector is:

$$dL(x, \lambda) = \frac{d(x, \lambda)e^{-\beta(\lambda)x}}{d\omega x^2} = k\beta(\lambda)e^{-\beta(\lambda)x}dx$$
 (Formula 2.5)

Integrating from x = 0 to d yields, can get the total atmospheric light intensity value:

$$L(d, \lambda) = k(1 - e^{-\beta(\lambda)d})$$
 (Formula 2.6)

The total light intensity received by the detector can be obtained by combining the sum of the reflected light from the target in the first step (points) and the scattered light from the atmosphere in the second step:

$$l(x) = \frac{L_{\infty}\rho(x)}{d^2} e^{-\beta(\lambda)d} + k(1 - e^{-\beta(\lambda)d})$$
(Formula 2.7)

## **III. OPTIMIZATION METHODS**

The Atmospheric Scattering Model obtained above is subject to certain errors, because the k in the equation represents the light source path, which is often regarded as a constant for the convenience of calculation and use, however, in fact in the vertical photographic path, especially when the photographic path passes through the mist, the light source path is not constant [4]. This paper uses three decay methods to improve it.

As for the Mile Scattering, the light source constant k\_m is calculated by the following Formula 3.1:

$$k_m = (\frac{\lambda^2}{4}) * D^2 * Q_ext$$
 (Formula 3.1)

 $\lambda$  stands for the wavelength, *D* stands for particle diameter, Q\_ext stands for is the extinction coefficient of the particle, which indicates the absorption and scattering ability of the particle to the incident light.

As for the Rayleigh Scattering, the light source constant k\_r is calculated by the following Formula 3.2:

 $k_r = (\frac{\lambda^4}{18}) * D^6 * Q_ext$  (Formula 3.2)  $\lambda$  stands for the wavelength, D stands for particle diameter, Q\_ext stands for is the extinction coefficient of the particle, which indicates the absorption and scattering ability of the particle to the incident light.

#### 1. Linear Decay Optimization

The decay of learning rate in machine learning is commonly used in a linear decay model, the decay process is linearly decreasing. Combined in the atmospheric scattering model, the distance between and point source can be specifically represented as the following, Formula 3.3 stands for the Mile Scattering and Formula 3.4 stands for the Rayleigh Scattering of the light source constant k m associated with the linear decay combination:

$$\begin{aligned} &k_m = (1 - tx) * \left(\frac{\lambda^2}{4}\right) * D^2 * Q_e xt \text{ (Formula 3.3)} \\ &k_r = (1 - tx) * \left(\frac{\lambda^4}{18}\right) * D^6 * Q_e xt \text{ (Formula 3.4)} \end{aligned}$$

x represents the distance from the point source and t represents the decay factor of the light source constant, t is set by different application scenarios. The Figure 3.1 shows the trend of the Linear Decay Light Source Constant.



After integrating from x=0 to d, we can get the new improved total atmospheric light intensity value for Mile and Rayleigh Scattering in Formula 3.5, 3.6 and the improved Atmospheric Scattering Model in Formula 3.7, 3.8 in the following:

$$\begin{split} \mathrm{L}(\mathrm{d},\lambda) &= \left(\frac{\lambda^2}{4}\right) * D^2 * Q_{-}ext * \left((\mathrm{td}-1+\frac{t}{\beta(\lambda)})e^{-d\beta(\lambda)}+1-\frac{t}{\beta(\lambda)}\right) \text{ (Formula 3.5)} \\ \mathrm{L}(\mathrm{d},\lambda) &= \left(\frac{\lambda^4}{18}\right) * D^6 * Q_{-}ext * \left((\mathrm{td}-1+\frac{t}{\beta(\lambda)})e^{-d\beta(\lambda)}+1-\frac{t}{\beta(\lambda)}\right) \text{ (Formula 3.6)} \\ \mathrm{I}(\mathrm{x}) &= \frac{L_{\infty}\rho(x)}{d^2}e^{-\beta(\lambda)d} + \left(\frac{\lambda^2}{4}\right) * D^2 * Q_{-}ext * \left((\mathrm{td}-1+\frac{t}{\beta(\lambda)})e^{-d\beta(\lambda)}+1-\frac{t}{\beta(\lambda)}\right) \text{ (Formula 3.7)} \\ \mathrm{I}(\mathrm{x}) &= \frac{L_{\infty}\rho(x)}{d^2}e^{-\beta(\lambda)d} + \left(\frac{\lambda^4}{18}\right) * D^6 * Q_{-}ext * \left((\mathrm{td}-1+\frac{t}{\beta(\lambda)})e^{-d\beta(\lambda)}+1-\frac{t}{\beta(\lambda)}\right) \text{ (Formula 3.8)} \end{split}$$

#### 2. Exponential Decay Optimization

Exponential decay is a decay method of machine learning rate, and the effect of applying exponential decay in some specific scenes will be better than linear decay, while the atmospheric scattering model light source constant changes with the distance from the point source can also be optimized by exponential decay. Formula 3.9 and 3.10 are the light intensity equations for Mile scattering and Rayleigh scattering combined with exponential decay, respectively.

$$k_m = (1 - x^t) * \left(\frac{\lambda^2}{4}\right) * D^2 * Q_e xt \text{ (Formula 3.9)}$$
  
$$k_r = (1 - x^t) * \left(\frac{\lambda^4}{18}\right) * D^6 * Q_e xt \text{ (Formula 3.10)}$$

x represents the distance from the point source and t represents the decay factor of the light source constant, t is set by different application scenarios. The Figure 3.2 shows the trend of the Exponential Decay Light Source Constant.



After integrating from x=0 to d, we can get the new improved total atmospheric light intensity value for Mile and Rayleigh Scattering in Formula 3.11, 3.12 and the improved Atmospheric Scattering Model in Formula 3.13, 3.14 in the following:

$$L(d, \lambda) = (\frac{\lambda^{2}}{4}) * D^{2} * Q_{ext} * (1 - e^{-\beta(\lambda)d} - \frac{e^{-\beta(\lambda)d}}{(-\beta(\lambda))^{t}} * \sum_{i=0}^{i=t} \frac{(-1)^{i} * t!}{(t-i)!} * (-d\beta(\lambda))^{t-i})$$
(Formula 3.11)  

$$L(d, \lambda) = (\frac{\lambda^{4}}{18}) * D^{6} * Q_{ext} * (1 - e^{-\beta(\lambda)d} - \frac{e^{-\beta(\lambda)d}}{(-\beta(\lambda))^{t}} * \sum_{i=0}^{i=t} \frac{(-1)^{i} * t!}{(t-i)!} * (-d\beta(\lambda))^{t-i})$$
(Formula 3.12)  

$$l(x) = \frac{L_{\infty}\rho(x)}{d^{2}}e^{-\beta(\lambda)d} + (\frac{\lambda^{2}}{4}) * D^{2} * Q_{ext} * (1 - e^{-\beta(\lambda)d} - \frac{e^{-\beta(\lambda)d}}{(-\beta(\lambda))^{t}} * \sum_{i=0}^{i=t} \frac{(-1)^{i} * t!}{(t-i)!} * (-d\beta(\lambda))^{t-i})$$
(Formula 3.13)  

$$l(x) = \frac{L_{\infty}\rho(x)}{d^{2}}e^{-\beta(\lambda)d} + (\frac{\lambda^{4}}{18}) * D^{6} * Q_{ext} * (1 - e^{-\beta(\lambda)d} - \frac{e^{-\beta(\lambda)d}}{(-\beta(\lambda))^{t}} * \sum_{i=0}^{i=t} \frac{(-1)^{i} * t!}{(t-i)!} * (-d\beta(\lambda))^{t-i})$$
(Formula 3.14)

#### 3. Discounted Future Reward Optimization

In considering the atmospheric scattering estimation part of the Atmospheric Scattering Model, when vertical photography, the light source constant is not a constant, in addition to the above two methods, we propose that we can also combine the Discounted Future Reward in deep learning to improve the adaptability of the atmospheric scattering model to the variation of the light source with distance points in vertical photography [5], the procedure is shown as the Figure 3.3. In order to perform equally well in a game with a long cycle, we need to consider not only the immediate rewards of the moment, but also what we can get in the future. For the atmospheric scattering model the variation of the light source constant in vertical photography is the same, and assuming the application of a Markov decision process [6], we can simply calculate the total attenuation R of the light source constant over a distance from the point source as follows Formula 3.5:







 $r_1 \dots r_n$  represents for the equally spaced distances from point light sources. However, because of the random nature of the environment, we cannot be sure if we will get the same decay next time with the same action. The more distant the distance, the greater the uncertainty. Therefore, it is common practice to use a decaying future instead of a deterministic one:

$$R_{t} = r_{t} + \gamma r_{t+1} + \gamma^{2} r_{t+2} + \gamma^{3} r_{t+3} + \dots + \gamma^{n-t} r_{n} = \frac{x}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t})$$
(Formula 3.16)

 $R_t$  is the decay value of the light source constant with distance,  $\gamma$  is a value between 0 and 1 to represent the attenuation factor with increasing distance. We can get the following:

 $R_t = r_t + \gamma(r_{t+1} + \gamma(r_{t+2} + ...)) = r_t + \gamma R_{t+1}$  (Formula 3.17)

Then we can get the light intensity equations for Mile scattering and Rayleigh scattering combined with exponential decay, respectively in Formula 3.18 and Formula 3.19.

$$k_m = (1 - (\frac{x}{n} * (1 + \gamma + \gamma^2 + \gamma^3 + ... + \gamma^{n-t}))) * (\frac{\lambda^2}{4}) * D^2 * Q_ext \text{ (Formula 3.18)}$$
  
$$k_r = (1 - (\frac{x}{n} * (1 + \gamma + \gamma^2 + \gamma^3 + ... + \gamma^{n-t}))) * (\frac{\lambda^4}{18}) * D^6 * Q_ext \text{ (Formula 3.19)}$$

After integrating from x=0 to d, we can get the new improved total atmospheric light intensity value for Mile and Rayleigh Scattering in Formula 3.20, 3.21 and the improved Atmospheric Scattering Model in Formula 3.22, 3.23 in the following:

$$\begin{split} \mathsf{L}(\mathsf{d},\lambda) = & (\frac{\lambda^{2}}{4}) * D^{2} * Q_{-}ext * (((\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t})) * \mathsf{d} - 1 + \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) e^{-d\beta(\lambda)} + \\ & 1 - \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) (\mathbf{Formula 3.20}) \\ \mathsf{L}(\mathsf{d},\lambda) = & (\frac{\lambda^{4}}{18}) * D^{6} * Q_{-}ext * (((\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))) * \mathsf{d} - 1 + \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) e^{-d\beta(\lambda)} + \\ & 1 - \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) (\mathbf{Formula 3.21}) \\ \mathsf{l}(\mathsf{x}) = & \frac{\mathcal{L}_{\infty}\rho(x)}{d^{2}} e^{-\beta(\lambda)d} + (\frac{\lambda^{2}}{4}) * D^{2} * Q_{-}ext * (((\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))) * \mathsf{d} - 1 + \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) e^{-d\beta(\lambda)} + 1 - \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) (\mathbf{Formula 3.22}) \\ \mathsf{l}(\mathsf{x}) = & \frac{\mathcal{L}_{\infty}\rho(x)}{d^{2}} e^{-\beta(\lambda)d} + (\frac{\lambda^{4}}{18}) * D^{6} * Q_{-}ext * (((\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))) * \mathsf{d} - 1 + \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) e^{-d\beta(\lambda)} + 1 - \frac{(\frac{1}{n} * (1 + \gamma + \gamma^{2} + \gamma^{3} + \dots + \gamma^{n-t}))}{\beta(\lambda)}) (\mathbf{Formula 3.23}) \end{aligned}$$

#### **IV. CONCLUSION**

The Atmospheric Scattering Model sets the light source constant to be invariant in the case of vertical photography, and the value changes with the distance from the point source in a particular case [7]. In this paper, we combine the atmospheric scattering model with the decay variation of the learning rate in machine learning and the feedback reward in Markov decision making process in deep learning, then we propose a combination of linear decay, exponential decay, and discounted future reward, three methods to optimize the light source constant of the atmospheric scattering model in the case of vertical photography. Although the traditional method of setting the light source constant as constant facilitates a lot of computational processing, in some cases of vertical photography [8], this will bring a lot of deviations, and to obtain accurate total light

intensity, the three optimizations proposed in this paper can further improve the accuracy, and the three optimizations are chosen according to the specific situation.

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